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Maximin Strategies and Elicitation of Preferences

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Introductory Remarks

It is shown below that the weakening of general demand requirements does not widen the class of satisfactory mechanisms in an essential way. And then, I can examine the case of discrete public projects, costing nothing to undertake, when utilities are separable additive. Each individual sends a message m_i to the center, chosen in a message space M_i, knowing that the project will be undertaken if a decision function $F(m)$, from X_iM_i to R takes on a positive value, and not undertaken otherwise. Side−payments $f_i(m)$ and $g_i(m)$ are associated with each occurrence. Agent i's payoff is : $f_i(m) + v_i$ if $F(m) > 0$, $g_i(m)$ if $F(m) \leqq 0$.

It is important to notice that A_i's maximin message is a function of v_i alone if $m_i = m_i(v_i)$。Optimality achieves if $\mathrm{sg}F(m_1(v_1), m_2(v_2), \ldots, m_n(v_n)) = \mathrm{sgx}\Sigma\, v_i \ \forall v$, where sgx is the sign of x.

Method

＊An Elicitation Scheme ―――――― :

$M = \{ F, (f_i, g_i), i = 1, n \}$ is constituted of n message spaces M_i, a decision function F from X_iM_i to R and n parts of side-payment functions (f_i, g_i) from X_iM_i to R such that the vector of maximin messages leads to the optimal decision. To simplify the analysis, it will be assumed that $M_i = R$ for all i, and that all the functions involved in the definition of the mechanisms are continuous. The continuity property is inheri−ted by the m_i. Now I can undertake to characterize elicitation schemes. It is necessary that, for every m_i, the sets $H^+(m_i) = \{ m_{-i} \varepsilon R^{n-1} / F(m_i, m_{-1}) > 0 \}$ and $H^-(m_i) = \{ m_{-i} \varepsilon R^{n-1} / F(m_i, m_{-i}) \leqq 0 \}$ be non−empty. Otherwise, the ith agent would sometimes impose the decision. Then, given F, f_i, and g_i, the ith individual computes, for every m_i :

$$(1)\ \begin{cases} \tilde{f}_i(m_i) = \underset{m_{-i}}{\mathrm{MIN}} f_i(m) & \text{s.t. } m_{-i}\ \varepsilon\ H^+(m_i) \\ \tilde{g}_i(m_i) = \underset{m_{-i}}{\mathrm{MIN}} g_i(m) & \text{s.t. } m_{-i}\ \varepsilon\ H^-(m_i). \end{cases}$$

Agent i then compares $\tilde{f}_i(m_i) + v_i$ and $\tilde{g}_i(m_i)$ and chooses m_i maximizing the smallest of those two numbers.

A first requirement on the $m_i(\cdot)$ is that they should be monotone : if $v_i \neq \bar{v}_i$, then $m_i(v_i) \neq m_i(\bar{v}_i)$. Otherwise, there would exist, for some i, two different values v_i^- and v_i^+ (with $v_i^- <$

v_i^+) associated with the same maximin message. Choosing v_j, $j \neq i$, so that $v_i^- + \sum_{j \neq i} v_j > 0$ and $v_i + \sum_{j \neq i} v_j > 0$, the n tuples of message $\{ m_i(v_i^+), m_j(v_j), j \neq i \}$ and $\{ m_i(v_i^-), m_j(v_j), j \neq i \}$ would be identical and would therefore lead to the same decision, which clearly violates optimality. Since $M_i = R$, and the m_i are continuous, they have to be monotone. In this case, given any increasing function $m_i(\cdot)$, the class of pair of side-payment functions f_i and g_i eliciting $m_i(v_i)$ as the unique maximin message of an individual whose true valuation is v_i, is characterized by :

$$(2)\begin{cases} \tilde{f}_i & \text{decreases} \\ \tilde{g}_i & \text{increases} \\ \tilde{f}_i(m_i(v_i)) + v_i = \tilde{g}_i(m_i(v_i)). \end{cases}$$

If m_i is a decreasing functuon, the above monotonicity conditions should be reversed. For example, if $m_i(\cdot)$ is assumed differentiable as well as increasing, a function $\tilde{g}_i^1(\cdot)$ can be selected so as to satisfy $0 < \tilde{g}_i^1(m_i(v_i)) < 1/m_i^1(v_i)$ for all v_i. Integration gives $\tilde{g}_i$ and equation (2) yields $\tilde{f}_i$. It is then easy to check that $\tilde{f}_i$ is decreasing. A similar construction could be carried out for decreasing $m_i(\cdot)$. In what follows, only increasing $m_i(\cdot)$ are considered.

The next step is to find what condition F should satisfy to guarantee the existence of increasing functions $m_i(\cdot)$ such that $sgF(m_1(v_1)), \ldots, m_n(v_n)) = sgx\ \Sigma v_i$.

A necessary condition is that the equation $F(m) = 0$ can be written in a separable additive form. This results from the following change of variables, made possible by the strict monotonicity of $m_i(\cdot)$, Call $\varphi_i = m_i^{-1}$. Then $v_i = \varphi_i(m_i)$.

And $F(m_1(v_1), \ldots, m_n(v_n)) = 0 \Leftrightarrow \Sigma\varphi_i(m_i) = 0$.

In addition, F should be increasing in all of its arguments at a point where it is equal to 0. Such a point m^* corresponds to a v^* at which $\Sigma v_i = 0$. By increasing any of the v_i^*'s, the sum of the v_i^*'s is made strictly positive and so shows F. Since $m_i(\cdot)$ is increasing, this implies that F has that property. Then, if $F(m) > 0$ (resp. $\leqq 0$), it is because $\Sigma v_i > 0$ (resp. $\leqq 0$). Suppose this is not the case : there exists $m^* = m(v^*)$ such that $F(m^*)$ is strictly negative (say) while Σv_i^* is positive. Take $v + \lambda e$ where e is the unit vector of R^n, and decreasing from 0 is a continuous way until the first 0 is reached, for some λ^*. At this point, $v + \lambda^* e$, F goes from negative to positive values, which contradicts the fact that it is supposed to increase in all of its arguments, as was stated in the above-mentioned.

Conversely, suppose that F is such that :

$F(m) = 0 \Leftrightarrow \Sigma\varphi_i(m_i) = 0$ for some φ_i, $i = 1, \ldots, n$. And also F(m) is increasing in all of its arguments at points where it is equal to 0. Then there exist functions $m_i(\cdot)$ such that $F(m_1(v_1), \ldots, m_n(v_n)) = 0 \Leftrightarrow \Sigma v_i = 0$.

The second condition imposed on F implies that the φ_i's are increasing. Their inverse is well defind. Choose the function $m_i(\cdot)$ to be φ_i^{-1}. I next show that it is essentially the only way to pick the functions $m_i(\cdot)$.

Call $\varphi_i(m_i(\cdot)) = w_i(\cdot)$. I am looking for functions $w_i(\cdot)$'s such that $w_i(v_i) = 0 \Leftrightarrow \Sigma v_i = 0$. Two cases should be distinguished :

(a) If the number of agents is strictly greater than 2, the functions $w_i(\cdot)$ should be all linear with equal slopes. * * *

Pick $v_K = 0$ for $i \neq k \neq j$. Then, $w_i(v_i) + w_j(v_j) + \sum_{i \neq k \neq j} w_K(0) = 0 \Leftrightarrow v_i + v_j = 0$. This implies that

(3) $w_i(v) + w_j(-v) - w_i(0) - w_j(0) = 0$ for all v. Since $\Sigma\, w_i(0) = 0$. This equation can be written for the couple(j, l), replacing v by $-$ v.

(4) $w_j(-v) + w_l(v) - w_j(0) - w_l(0) = 0$ for all v, and for (1, i).

(5) $w_l(v) + w_i(-v) - w_l(0) - w_i(0) = 0$ for all v. Multiplying the second of those three equations by -1 and adding them gives :

(6) $w_i(v) + w_i(-v) - 2\,w_i(0) = 0$ for all v, and for all i. Combining the equation (3) and equation (6) yields :

(7) $w_i(v) - w_i(0) = w_j(v) - w_j(0)$ for all i and for all j. Next, given v_i, suppose that $v_i = -(v_i / n-1)$ for all $j \neq i$. Since $\Sigma\, v_l = 0$,

(8) $w_i(v_i) + \sum_{j \neq i} (w_j(0) - w_i(0)) + (n-1) w_i(-v_i/(n-1)) = 0$, for all $v_i = w_i(v_i) - nw_i(0) - (n-1) w_i(-v_i/(n-1)) = 0$. From the equation (6) and the equation (7), $w_i(v_i) = nw_i(0) - (n-1)\, 2\,w_i(0) - w_i(v_i/(n-1))$.

(9) $w_i(v_i) = (2n-n) w_i(0) + (n-1)\, w_i(v_i/(n-1))$. The only solutions to the equation (9) are of the form $w_i(v_i) = av_i + b_i$ where $b_i = w_i(0)$. Then, from the equation (7), $w_j(v_j) = av_j + b_j$, and $m_j(v_j) = \varphi_j^{-1}(av_j + b_j)$, with the b_j's adding up to 0.

(b) The second case is when the number of agents is equal to 2. Then, $w_1(\cdot)$ and $w_2(\cdot)$ should have their graphs symmetric with respect to the origin.

For instances : ———,

If n= 2, the equation (9) is identically satisfied. The only condition is (3) which can be written $w_1(v) + w_2(v) = 0$ for all v (since $w_1(0) + w_2(0) = 0$).

Verifications

(PROPOSITION I)

"In order for a function $F(\cdot)$ to be an admissible decision function it should be such that $F(m) = 0 \Leftrightarrow \Sigma \varphi_i(m_i) = 0$ for some monotone φ_l's. Then, the only way to select response functions $m_i(\cdot)$ is as follows :

[1] if the number of agents is strictly greater than 2, $m_i(v_i) = \varphi_i^{-1}(av_i + b_i)$ with a $\neq$

0 $\Sigma b_i = 0$.

〔2〕 if the number of agents is equal to 2, $m_i(v_i) = \varphi_i^{-1}(h_i(v_i))$ where h_1 and h_2 are monotone functions, the graphs of which are symmetric with respect to the origin. Finally, given any monotone $m_i(\cdot)$, there are an infinity of ways to select monotone $\tilde{f}_i$ and $\tilde{g}_i$ such that, confronted to the scheme F, f_i, g_i, the ith agent will find $m_i(v_i)$ as his unique maxi−min strategy. The class of such functions is given by the equation (1).

Such schemes do not constitute as essential widening of the class of elicitation mechanisms. "

* Remarks : * * * * * * *

So far, it has been assumed that each agent's strategy space was the whole real line.

However, situations exist in which it is known that the project is a public good for all agents, so that the center can in such cases legitimately restrict announced valuations to be non−negative. Then a public good that costs nothing to produce should always be undertaken, so that it is important to explicitly introduce the cost C of the public good, where C is a strictly positive number. Consider the scheme under which each agent pays what he announces. That is to say : ――――――― The truth is one of the maximin strategies when true valuations can be either positive or negative. In the situation we are examining now, an individual whose true valuation happens to be strictly greater than C would not announce the truth as a maximin strategy since a strategy of $C + \varepsilon$ would lead him to a strictly positive gain with certainty, while the truth would lead him to a zero utility also with certainty. Notice, however, that such misrepresentation does not destroy optimality. And all the elicitation schemes have that property. Truth will not be forthcoming from an agent whose true valuation exceeds the cost of the project. However, opimality would not be violated by such misrepresentation.

Let us now turn the determination of the optimal size of a public project.

Each utility function depends on a parameter θ_i known to agent i alone. We impose the additional requirement that θ_i be a non−negative real number. Utilities are again assumed to be separable and linear in money. In the first step, we will not demand that the budget of the center be balanced. More precisely, we make the following assumptions :

* Assumptions : * * * * * * *

Al : "$u_i(\theta_i ; a_i, y) = a_i + v_i(\theta_i ; y)$, where a_i is the amount of money held by A_i and y is the level of the public good. The function v is twice differentiable, and is an increasing concave function of y. Formally : $(\partial v / \partial y) \cdot (\theta_i ; y) \geqq 0$, $(\partial^2 v / \partial y^2) \cdot (\theta_i ; y) \leqq 0 \ \forall \theta_i, y$. In addition, $(\partial v / \partial \theta) \cdot (\theta_i ; y) \geqq 0$, $(\partial^2 v / \partial y \partial \theta) \cdot (\theta_i, y) \geqq 0$, $v(0 ; y) = 0 \ \forall \theta_{ii}, y$.

Without loss of generality, initial holdings of money are taken to be equal to 0. "

A 2 : "The production technology of the public good is represented by a function $x = g(y)$, where x is the input necessary for the production of an amount y of the public good. $g(\cdot)$ is differentiable three times and its derivatives satisfy : $g'(\cdot) \geqq 0$, $g''(\cdot) \geqq 0$."

This last condition indicates decreasing returns in the production of the public good. The revelation game takes the following form : each consumer announces a value of his parameter θ_i, chosen in his strategy space $S_i = R^+$, possibly different from his true value $\mathring{\theta}_i$, denoted by a circle. The center computes the optimal level of the public good corresponding to those announced values, $y(\theta)$, assuming those announcements to be true, by solving in y the Samuelson optimality condition.

$$(10)\quad \Sigma \frac{\partial v(\theta_i;y)}{\partial y} = g'(y).$$

The center also distributes side−payments $f_i(\theta)$ to all the agents. Because of the separability of the utility functions, this procedure is legitimate since the optimal level of the public good does not depend on the distribution of money in the economy. The ith agent's payoff is then $V(\mathring{\theta}_i ; \theta) = f_i(\theta) + v(\mathring{\theta}_i ; y(\theta))$. I can consider the maximin strategies of this game, and investigate the conditions under which they lead to the choice of the optimally, denoted $y(\mathring{\theta})$. Later on, I can impose the further requirement that the budget of the center be balanced :

$$(11)\quad \Sigma f_i(\theta) = -g(y(\theta)).$$

An elicitation scheme is a n−tuple($f_i(\theta)$, $i = 1, \ldots, n$) enjoying all of the above properties.

(PROPOSITION Ⅱ)

"If the budget balanced condition is not imposed, there are tax functions such that announcing one's true parameter is a maximin strategy."

＊Remarks : ＊＊＊＊＊＊＊

From the work of Groves and Loeb, I can know that there exist tax functions for which it is a dominant strategy to announce one's true parameter. An example of such a mechanism is given by $f_i(\theta) = \sum_{i \neq j} v(\theta_j ; y(\theta)) - g(y(\theta) - [\sum_{i \neq j} v(\theta_j ; y(\theta_{-i})) - g(y(\theta_{-1}))]$, where $y(\theta)$ and $y(\theta_{-i})$ respectively represent the optimal levels of the public good when all the agents are present, and when the ith agent has been deleted from the economy. This mechanism is just one specification of a general class. No element of this class can satisfy the budget balance condition, though, as was established by Green and Laffont. This is what motivates us to look for other solutions. To start with, and because it simplifies the analysis, I can investigate the existence of tax functions for which the minimization of the ith individual's utility in θ_j, for j different from i, is obtained at the corner of A_j's strategy space, $\theta_j = 0$.

(PROPOSITION Ⅲ)

"If for every θ_i and $\mathring{\theta}_i$ elements of R^+, the minimization problem admits of the solution $\theta_j = 0$, for $j \neq i$, then $f_i(\theta)$ should satisfy : $f_i(\theta_i, 0) = -g(y(\theta_i, 0)) + k$, where $y(\theta_i, 0)$ is the optimal level of the public good corresponding to the vector of announced valuations(θ_i, 0) (which denotes $(0, \theta_i, 0, \ldots, 0)$ with a slight abuse of notation), and k is an arbitrary constant."

＊Proof : ＊＊＊＊＊＊＊

Minimization of A_i's indirect utility in θ_j, $j \neq i$, yields a new function $W(\mathring{\theta}_i ; 0_i) = v(\mathring{\theta}_i ; \theta_i, 0)$ that depends only on $\mathring{\theta}_i$ and θ_i. $W(\mathring{\theta}_i ; \theta_i) = f_i(\theta_i, 0) + v(\mathring{\theta}_i, y(\theta_i, 0))$. It is this function which is maximized in θ_i by A_i, giving $\{(\partial W(\mathring{\theta}_i ; \theta_i))/(\partial \partial_i)\} = \{\partial f_i(\mathring{\theta}_i, 0)/\partial\theta_i\} + [\{\partial v(\mathring{\theta}_i ; y(\theta_i, 0))/\partial y\} * \{\partial y(\theta_i, 0)/\partial\theta_i\}] = 0$. In order for the solution of this equation in θ_i to have the solution $\theta_i = \mathring{\theta}_i$, no matter what $\mathring{\theta}_i$ is, it is necessary that this expression be an identity in $\mathring{\theta}_i$, when θ_i is replaced by $\mathring{\theta}_i$. This identity defines $f_i(\theta_i, 0)$. Dropping the circle over θ_i for simplicity of notation and rewriting the Samuelson condition for the case $\theta_j = 0$, $j \neq i$, gives $(\partial v/\partial y)\cdot(\theta_i ; y(\theta_i, 0)) = g'(y(\theta_i, 0))$ so that $(\partial f_i/\partial\theta_i)\cdot(\theta_i, 0) + g'(y(\theta_i, 0))(\partial y/\partial\theta_i)\cdot(\theta_i, 0) \equiv 0$. Integration in θ_i yields $f_i(\theta_i, 0) + g(y(\theta_i, 0)) + k = 0$, which is the condition stated in the above-shown proposition. Its integration is simple. It means that if all agents but A_i want none of the public good, the remaining agent should bear the full cost of its implementation. Free-riding is then impossible. It remains to show that the extremum obtained is indeed a maximum. From the definition of $f_i(\theta_i, 0)$; the following expression is identically equal to 0.

$$(12)\quad \frac{\partial w(\theta_i;\theta_i)}{\partial\theta_i} = \frac{w(\theta_i;o_i)}{\partial\theta_i}\Bigg|_{\mathring{\theta}_i = o_i}$$

$$\frac{\partial w(\theta_i;\mathring{\theta}_i)}{\partial\mathring{\theta}_i}\Bigg|_{\theta_i = \mathring{\theta}_1} \equiv 0.$$

So that, differentiating once more :

$$(13)\quad \frac{\partial^2 w(\mathring{\theta}_i;\theta_i)}{\partial\theta_i^2}\Bigg|_{\mathring{\theta}_i = \theta_i} = -\frac{\partial^2 w(\mathring{\theta}_i;\theta_i)}{\partial\mathring{\theta}_i}\Bigg|_{\mathring{\theta}_i = \theta_i} =$$

$$-\frac{\partial^2 \nu(\theta_i;y(\theta_i,0))}{\partial y\partial\theta_i} * \frac{\partial y(\theta_i,0)}{\partial\theta_i}$$

The second term of this product is given by differentiation of the Samuelson condition :

$$(14)\quad (\partial y(\theta)/\partial\theta_i) = \frac{(\partial^2 v/\partial y\partial\theta_i) * (\theta_i;y(\theta))}{g''(y(\theta)) - \Sigma(\partial^2/\partial y^2)(\theta_i;y(\theta))}$$

which is positive for all θ, as A 1 and A 2 guarantee. Since $(\partial^2 v/\partial y\partial\theta)$ is positive (by A 1), it follows that the second order derivative of the maximand is negative.

*Examples : * * * * * * *

$v(\theta ; y) = \theta \mathrm{Log} y$. It is easy to verify that A 1 and A 2 are satisfied. The Samuelson condition takes the form : $\Sigma\theta_i = y(\theta)g'(y(\theta))$.

ⓐ —— With a linear technolgy $g(y) = y$, it becomes $\Sigma\theta_i = y(\theta)$ and therefore $\theta_i = y(\theta_i, 0)$. The tax function $f_i(\theta_i, 0)$ is then $f_i(\theta_i, 0) = -g(y(\theta_i, 0)) + k = -\theta_i + k$.

ⓑ —— With a more general technology of the form $g(y) = y^e$, similar computations would give $f_i(\theta_i, 0) = -A(e)\theta_i^{B(e)} + k$, where A(e) and B(e) are algebraic expressions depending on e.

Now, I can impose the additional requirement that the budget of the center be balanced.

(PROPOSITION IV)

"A necessary and sufficient condition for the existence of a balanced elicitation scheme for which minimization in θ_j, $j \neq i$, takes place at the corners ($\theta_j = 0$) is that

$$g(y(\theta)) \leqq \Sigma g(y(\theta_i, 0)) \text{ for all } \theta.\text{"} \tag{15}$$

This condition tells us how the curvatures of the utility and production functions should be related. Before proving this proposition, I introduce a definition.

*Definition : * * * * * * *

A set of n functions $\varepsilon_i(\theta)$ from R^{+n} to R^+ are called sharing rules iff

1. ——— $\Sigma\varepsilon_i(\theta) = 1$ for all θ such that $\sum_{j=1}^{m}\theta_j \neq 0$.
2. ——— $\varepsilon_i(\theta_i, 0) = 0$ for all θ_i.

An example of such functions is.

$$(16)\quad \begin{cases} \varepsilon_i(\theta) = \sum_{j\neq i}\theta_j/(n_{-1})\sum_{j=1}^{m}\theta_j & \text{if } \sum_{j=1}^{m}\theta_j \neq 0 \\ \\ \varepsilon_i(\theta) = 0 & \text{if } \sum_{j=1}^{m}\theta_j = 0 \end{cases}$$

*Proof : * * * * * * *

In order for minimization in θ_j, $j \neq i$, to be achieved at $\theta_j = 0$, it should be the case that : $f_i(\theta_i, \theta_j) + v(\mathring{\theta}_i ; y(\theta_i, \theta_j)) \geqq f_i(\theta_i, 0) + v(\mathring{\theta}_i ; y(\theta, 0))$ $\forall \mathring{\theta}_i, \theta_i, \theta_j$.
Since $y(\theta_i, \theta_j) \geqq y(\theta_i, 0)$ for all θ_j, and since $v(\mathring{\theta}_i, y)$ is a non−decreasing function of y, it follows that the inequality will certainly hold if $f_i(\theta_i, \theta_j) \geqq f_i(\theta_i, 0)$ $\forall\theta_i$, $\forall\theta_j$ .... This is also a necessary condition. Indeed, for $\mathring{\theta}_i = 0$, the above-shown equation reduces to $f_i(\theta_i, \theta_j) \geqq f_i(\theta_i, 0)$, $\forall\theta_i$, $\forall\theta_j$. Adding up across i.

(17) $\Sigma f_i(\theta_i, \theta_j) = -g(y(\theta)) \geqq \Sigma f_i(\theta_i, 0) = -\Sigma g(y(\theta_i, 0))$, the first inequality resulting from

the budget balance condition, the second from (15), and the last from Proposition IV. The condition given in the theorem is therefore sufficient. That it is also necessary is easily seen : if it did not hold, at least one equality of the above−shown would have to be violated. Given a set of sharing rules, the following functions define an elicitation scheme : $f_i(\theta) = -g(y(\theta_i, 0) + \varepsilon_i(\theta)\ [\Sigma g(y(\theta_i, 0)) - g(y(\theta))]$, $i = 1, \ldots, n$.

Since the expression in the brackets is always non−negative, and its coefficient $\varepsilon_i(\theta)$ also has that property, the second term of this tax function can be interpreted as a rebate to agent i, having paid $-g(y(\theta_i, 0))$. This second term only depends on θ_j, $j \neq i$, and its value is minimized at $\theta_j = 0$. The individual is then confronted with the tax function defined in Proposition I, which induces truthful revelation.

*Example a : * * * * * * *

$v(\theta, y)=\theta \mathrm{Log} y$, and $g(y)=y$. This is the example of the last proposition. Then, $g(y(\theta))=\Sigma\theta=\Sigma g(y(\theta_i, 0))$. The condition of Proposition IV is satisfied at equality. No rebate is actually necessary and budget balance achieves with $f(\theta)=-\theta_i$.

*Example b : ******

If $v(\theta, y)=\theta y^{0.5}$ and $g(y)=y$, simple computations lead to : $g(y(\theta))=0.25(\Sigma\theta)^2>0.25\Sigma\theta_i^2=\Sigma g(y(\theta_i, 0))$

*Example c : ******

If $v(\theta, y)=\theta y^{0.5}$ and $g(y)=0.67y^{1.5}$, $g(y(\theta))=(\Sigma\theta_i/2)=\Sigma(\theta_i/2)=\Sigma g(y(\theta_i, 0))$.

The last proposition deals with the case when condition (15) does not hold. Even though it is not possible to guarantee that minimization in θ_j is achieved at $\theta_j=0$, it is nevertheless possible to exhibit an elicitation scheme.

(PROPOSITION V)

"Given sharing rules $\varepsilon_i(\theta)$, the functions $f_i(\theta)=f_i^*(\theta)+\varepsilon_i(\theta)\cdot[-\Sigma f_i^*(\theta)-g(y(\theta))]$ where $f_i^*(\theta)=-v(\theta_i, y(\theta))-g(y(\theta_i, 0))+v(\theta_i, y(\theta_i, 0))$ constitute an elicitation scheme."

Concluding Remarks

It is easy to check that the budget balance condition holds. Next I can show that $h(\theta)=-\Sigma f_i^*(\theta)-g(y(\theta))\geqq 0$ for all θ. If $\theta=0$, then $y(0)=0$, $f_i^*(0)=0$, and $g(y(0))=0$. It follows that $h(\theta)=0$. In addition, h is differentiable and

(18) $$(\partial h(\theta)/\partial\theta_i)=\frac{+\partial v(\theta_i;y(\theta))+g'(y(\theta_i, 0)}{\partial\theta_i}*\frac{\partial y(\theta_i, 0)}{\partial\theta_i}-\frac{\partial v(\theta_i, y(\theta_i, 0))}{\partial\theta_i}-\frac{\partial v(\theta_i, y(\theta_i, 0))}{\partial y}*\frac{y(\theta_i, 0)}{\partial\theta_i}+\Sigma\frac{\partial v(\theta_i, y(\theta))}{\partial y}*\frac{\partial y-g'(y(\theta))}{\partial\theta_i}*\frac{\partial y}{\partial\theta_i}$$

The last two terms, after factoring out $\frac{\partial y}{\partial\theta_i}$, can be seen to add up to 0, by definition of $y(\theta)$. This is SAMUELSON condition. The second term and the fourth term, after factoring out $\frac{\partial y(\theta_i,0)}{\partial\theta_i}$, add up too,for the same reason.

Since $y(\theta)\geqq y(\theta_i, 0)$, the first and third term add up to a non−negative number, because of the cross−derivative assumption A 1. The partials of $h(\theta)$ are non−negative at every point and $h(\theta)=0$; therefore $h(\theta)$ is everywhere non−negative.

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収束型選好最適化モデルの検証＊＊（要約）

本稿では　まず，伝統的な行動主義仮説とは若干異なった仮説を新規につぎのような形で論定する。

(1)　：各主体は最適化モデルで特定されるリスク忌避の行動様式をとるものと仮定する。

(2)　：各主体が想定している各種の最適化モデルのうちのどれかひとつを特定したとき，当該主体は自分にとって極めて不利な構築内容を内臓したほかの主体の最適化モデルをえらび，この種のモデルにより規定される負の最大効用量に対応した形で当該主体自身に固有のモデル構築を作業する。

ところで，Drezeとde la Vallee－Poussinは，真の限界価値を規定できる最適化モデルの造成作業を逐一クリアできるという特性を内臓した動学的公共配分モデルの展開作業を試みた。本稿での私のネライは，この種の特性を内臓した配分メカニズムの特徴を検出することにある。従って，本稿での論証作業の内容は (i) Nashタイプの均衡モデルでクリアできる負の効用均衡量の存在証明，および (ii) 最適化行動モデルは，必ず，実用に耐えうる均衡モデルの存在条件を保証できるツールになるというコンセプトの確認
の二つである。

（付記：本稿は1989年７月，New England大学（Australia）での国際計量経済学会年次大会で研究報告した論文の第４－５章を全面的にかきなおしてまとめたものである。）