

Incomplete Forward Market Dynamics on Exchange Economic System

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Introduction

Complete contingent commodity markets provide a convenient framework in which problems involving choices under uncertainty can be analysed. Yet the absence of such complete markets has been noted by many authors. Among them, Jacques Dreze has stressed the need for research into the functions and shortcomings of existing institutions.¹⁾ The polemic and discussive view—points in this paper is to examine the workings and welfare implications of actual forward markets and to place those markets in the context of complete contingent commodity markets. In actual forward markets a typical forward contract is one which is binding independent of the state of the system at the time of maturity. Forward currency and commodity future contracts are examples of such unconditional contracts. So, concerning to the equilibrium with unconditional contract, I can examine the existence and properties of equilibria in models restricted exogenously to unconditional contracts. It is shown by way of some examples that risk averse participants use unconditional contracts to hedge against the randomness of both spot prices and their exogenous endowments; with active spot markets, optimal behaviour does not involve the elimination of risk by purchasing the consumption bundle forward. On the problem that all individuals are identical and homothetic, I provide a welfare analysis of unconditional contracts. It is shown that in a two—state economic system, unconditional contracts with active spot markets may achieve Pareto optimal allocations. Contrary to the comments by Radner,²⁾ subsequent spot markets need not introduce externalities which affect the optimality of competitive equilibria. With more states than two, the outcome with spot and unconditional contracts is in general Pareto non-optimal though, with identical and homothetic preferences, Pareto superior to the outcome with all forward contracts prohibited. Hence, future contracts do not usually compensate for the absence of complete contingent contracts, but they do make possible beneficial hedging by all parties. So, the second best government policies in the absence of complete markets are explored. The welfare analysis would lead one to expect more contingent contracts than are observed in actual markets. Though it is not the purpose of this paper to explain the absence of such contracts, on the problem that all individuals have constant and relative risk aversion, I offer some thoughts on the preponderance of unconditional contracts. If contracts are contingent on information not revealed to everyone, moral hazard would seem to require third party verification of states.

Unconditional contracts would seem to be costless relative to the certification schemes associated with contingent contracts.

Basic Structures of The Models

The model is a two-goods pure exchange economy with stochastic endowments. Before the random endowments are realized, each individual can decide on the quantity and type of forward contracts to purchase or issue. After endowments are realized, forward contracts are executed, and trade and consumption take place in spot markets. In the model no actual transfer or consumption of resources takes place in forward markets; only claims are traded. There is no uncertainty in spot markets; every individual knows his endowment and the market price. An individual's consumption is not limited to the claims acquired in forward market—retrading is possible in the spot markets. The exogenous endowment of each of m individuals is assumed to have a discrete probability distribution with n possible realizations. A state of the economic system specifies for each individual the realized endowment of each of the two goods (X) and (Y). Let Ω be the set of all possible states of the economic system with typical element w . Then, w is an m -dimensional row vector specifying a state for each individual. By construction Ω has n^m elements. Let (Ω, F, g) be a discrete probability space where F is the set of all subsets of Ω and $g(w)$ is the probability that state w will occur,

$$Z^i(w)' = \left[Z_x^i(w), Z_y^i(w) \right]$$

where $Z_j^i(w)$ is the i th individual's exogenous endowment of commodity (j) in state w . In the planning period each individual is assumed to maximize expected utility. Let $c_j^i(w)$ be the i th individual is assumed to maximize expected utility. Let $c_j^i(w)$ be the i th individual's consumption of commodity (j) in state w . Then, each individual acts as if to maximize

$$\sum_{w \in \Omega} g(w) U^i \left[c_x^i(w), c_y^i(w) \right].$$

Each individual subjective probability distribution of states of the economic system is the actual probability measure $g(\cdot)$ of the model. In this manner, $U^i(\cdot, \cdot)$ is the utility of individual i in state w . State dependent utility functions

$U^{i,w} [c_x^i(w), c_y^i(w)]$ could be assumed without affecting the analysis which follows. It is further assumed that

- (a) $U^i(\cdot, \cdot): \mathbb{R}^2 \rightarrow \mathbb{R}^1$ with continuous partial derivatives $U_1^i(\cdot, \cdot), U_2^i(\cdot, \cdot)$, strictly positive for $c_x^i(w) > 0, c_y^i(w) > 0$; and $U_1^i(0, 0) = \infty$.
- (b) $U^i(\cdot, \cdot)$ is strictly concave.

The properties of $U^i(\cdot, \cdot)$ are sufficient to ensure the existence of a competitive equilibrium in each state w . At this point, uniqueness of the competitive equilibrium may be taken as an assumption, though in the special cases discussed below, uniqueness follows from additional assumptions. Property (b) ensures that each

individual is risk averse.

The analysis is facilitated by the use of the indirect utility function.

Let $I^i(w)$ = the income of individual i in terms of (X) in state w

h_{xw}^i = the demand for good (X) in state w by individual i

h_{yw}^i = the demand for good (Y) in state w by individual i

$P(w)$ = the price of (Y) in terms of (X) in state w

$r(w)' = 1, P(w)$

Then, $[h_{xw}^i, h_{yw}^i]$ maximize $U^i [c_x^i(w), c_y^i(w)]$ subject to $I^i(w) = P(w) c_y^i(w) + c_x^i(w)$.

Define the indirect utility function V^i as $V^i [I^i(w), P(w)] = U^i \{h_{xw}^i [I^i(w), P(w)], h_{yw}^i [I^i(w), P(w)]\}$.

Complete forward markets are described as follows. Each individual can issue or purchase contingent commodity claims, each of which entitles the holder to one unit of the p specified commodity if a particular state of the economic system occurs.

Let $X^i(w)$ = claims on (X) contingent on state w acquired by individual i in the market for claims $Y^i(w)$ = claims on (Y) contingent on state w acquired by individual i

in the market for claims. Equilibrium in the market for claims exists when $\sum_{i=1}^m X^i(w) = 0$

and $\sum_{i=1}^m Y^i(w) = 0$ for each $w \in \Omega$. Let $P_y(w)$ = the price of one claim on one unit of (Y) in state w . As yet, the numeraire is unspecified. The budget in t for individual i in the market or claims is that the net value of excess demand for claims be zero, or

$$(1) \quad \sum_{w \in \Omega} [X^i(w) P_x(w) + Y^i(w) P_y(w)] = 0, i = 1, 2, \dots, m.$$

With insurance schemes an individual's income in state w is the value of his claims on (X) and on (Y) in state w , and the value of his exogenous endowment in state w .

$$(2) \quad I^i(w) = r(w)' Z^i(w) + P(w) Y^i(w) + X^i(w).$$

To prevent bankruptcy it is specified that $I^i(w) \geq 0$, for each $w \in \Omega$. However, by property (a) of $U^i(\cdot, \cdot)$, $V^i[0, P(w)] = +\infty$ for each $w \in \Omega$ and, therefore, the bankruptcy constraint will never be binding in equilibrium. In summary, the objective of individual i is to maximize

$$(3) \quad \sum_{w \in \Omega} g(w) V^i [r(w)' Z^i(w) + P(w) Y^i(w) + X^i(w), P(w)] - \lambda \left\{ \sum_{w \in \Omega} P_x(w) X^i(w) + P_y(w) Y^i(w) \right\}$$

with respect to $\{X^i(w), Y^i(w)', w \in \Omega\}$ where λ is a Lagrange multiplier. This yields first order condition (4).

Identity and Homotheticity in Preferences of all Individuals

Where unambiguous the superscript i on utility functions is now deleted. Under property in the polemic points concerned with the equilibrium with unconditional contracts, the spot price will depend on the ratio of the aggregate endowment of (X) to the aggregate endowment of (Y).

$P(w) = f \left[\frac{\sum_{i=1}^m Z_x^i(w)}{\sum_{i=1}^m Z_y^i(w)} \right]$, Hence, spot market prices will be independent of the existence and direction of forward contracts. Under property in the problem with regard to the equilibrium with unconditional contracts, some characteristics of the demand for unconditional forward contracts can be derived. Let

$$E = \{F : \min_{w \in \Omega} P(w) < F < \max_{w \in \Omega} P(w)\}$$

LEMMA 1 : "For each $F \in E$, $\sum_{w \in \Omega} V \{r(w)' Z(w) + Y_f^1 [P(w) - F], P(w)\}$ is strictly concave with respect to Y_f^1 ."

LEMMA 1 follows immediately from the property in the Basic Models and Structures of The Models, and the proof is not given here. (6) is continuous with respect to $P^1(w)$. Given $F \in E$, let $S_1 = \{w : F - P(w) > 0\}$ and $S_2 = \{w : F - P(w) < 0\}$. Then, from the bankruptcy constraints

$$\max_{w \in S_2} [r(w)' Z^1(w) / F - P(w)] \leq Y_f^1 \leq \min_{w \in S_1} [r(w)' Z^1(w) / F - P(w)].$$

Hence, for each $F \in E$, a maximizing choice of Y_f^1 does exist as a continuous function on a compact set must achieve a maximum on that set. By LEMMA 1, the choice must be unique. This optimum value of Y_f^1 will be denoted $Y_f^{1*} = \varphi^1(F)$. From (6), a necessary and sufficient condition for a maximum is that

$$(7) \quad \sum_{w \in \Omega} g(w) [P(w) - F] V_1 \{r(w)' Z^1(w) + Y_f^{1*} [P(w) - F], P(w)\} = 0.$$

Riskless arbitrage will ensure that Y_f^{1*} is infinite in absolute value if $F \in E$.

Hence, the search for a competitive equilibrium can be limited to $F \in E$.

LEMMA 2 : "Given properties that were shown in the above-mentioned polemic points of all, especially, concerned with $U(\cdot, \cdot)$, $\varphi^1(F)$ has continuous derivatives."

(PROOF of the LEMMA 2) : (As noted, $P^1(w)^* > 0$, $w \in \Omega$. Hence, Y_f^1 will always be an interior solution. Let

$$G^1(F, Y_f^1) = \sum_{w \in \Omega} g(w) [P(w) - F] V_1 \{r(w)' Z^1(w) + Y_f^1 [P(w) - F], P(w)\}.$$

Then, for each $F \in E$, $G^1[F, \varphi^1(F)] = 0$; $G_1^1(\cdot, \cdot)$, $G_2^1(\cdot, \cdot)$ are continuous with respect to F and Y_f^1 ; $G_2^1(\cdot, \cdot) < 0$ by LEMMA 1. Hence, the implicit function

theorem applies : $\min_{w \in \Omega} P(w) = P', \max_{w \in \Omega} P(w) = P''$.

LEMMA 3 : “Under properties that were shown in the above-mentioned polemic points of all, especially, concerned with $U(\cdot, \cdot)$, $\lim_{F \rightarrow P'} \varphi^1(F) = +\infty$ and $\lim_{F \rightarrow P''} \varphi^1(F) = -\infty$.”

(PROOF of the LEMMA 3) : (Suppose it were not the case that $\lim_{F \rightarrow P'} \varphi^1(F) = +\infty$.

Then, one can construct a sequence $\{F_n\}^{E \rightarrow P'}$ εE such that $\lim_{n \rightarrow \infty} F_n = P'$ and the corresponding sequence $\{\varphi^1(F_n)\}$ is always less than some positive number K . Then,

$$\begin{aligned} \lim_{F_n \rightarrow P'} \sum_{w \in \Omega} g(w) V_1 \{r(w)' Z^1(w) + \varphi^1(F_n) [P(w) - F_n], P(w) [P(w) - F_n]\} &\geq \\ \lim_{F_n \rightarrow P'} \sum_{w \in \Omega} g(w) V_1 \{r(w)' Z^1(w) + K [P(w) - F_n], P(w)\} [P(w) - F_n] &= \\ \sum_{w \in \Omega} g(w) V_1 \{r(w)' Z^1(w) + K [P(w) - P'], P(w)\} [P(w) - P'] &> 0 \end{aligned}$$

This is the desired contradiction as for each $F \varepsilon E$, $G^1[F, \varphi^1(f)] = 0$.

The proof that $\lim_{F \rightarrow P''} \varphi^1(F) = -\infty$ follows similarly.)

PROPOSITION I * * * * * :

“Under properties that were shown in the above-mentioned polemic points of all, especially, concerned with $U(\cdot, \cdot)$, the model restricted exogenously to unconditional forward contracts possesses a cometic equilibrium.”

(PROOF of the PROPOSITION I) : (Define $\varphi(F) = \sum_{i=1}^m \varphi^i(F)$. By LEMMA 2, $\varphi^1(F)$ is continuous on E . By LEMMA 3, there exist F' and F'' in E such that $\varphi(F') > 0$, $\varphi(F'') < 0$. Therefore, by the intermediate value theorem there exists some F^* such that $\varphi(F^*) = 0$. This establishes existence.)

Constant and Relative Risk Aversion of all Individuals

A coefficient of relative risk aversion, C_r , is defined in terms of the indirect utility function, i. e.,

$$C^1 = C^1 [I^1(w), P(w)] = -V_{11} [I^1(w), P(w)] I^1(w) / V_1 [I^1(w), P(w)].$$

Constancy means that C^1 is independent of both prices and incomes for all states. A coefficient of absolute risk aversion, D^1 , is defined as follows:

$$D^1 [I^1(w), P(w)] = -V_{11} [I^1(w), P(w)] / V_1 [I^1(w), P(w)]. \text{ Then, } D^1 [I^1(w), P(w)] = C^1 / I^1(w) \text{ and } D^1 \text{ is strictly decreasing in } I^1(w).$$

LEMMA 4 : “Given properties in the polemic points that were shown at the

above-mentioned, especially, concerned with $U(\cdot, \cdot)$, $\varphi^i(F)$ is strictly monotone decreasing.”

(PROOF of the LEMMA 4) : (By LEMMA 2, $\varphi^i(F)$ is differentiable, and $\{d\varphi^i(F) / dF\} = \{-G_1^i(F, Y_i^{i*}) / G_2^i(F, Y_i^{i*})\}$, $G_1^i(F, Y_i^{i*}) = -Y_i^{i*} \sum_{w \in \Omega} g(w) [P(w) - F] V_{11} [I^i(w)^*, P(w)] - \sum_{w \in \Omega} g(w) V_1 [I^i(w)^*, P(w)]$, $G_2^i(F, Y_i^{i*}) = \sum_{w \in \Omega} g(w) [P(w) - F]^2 V_{11} [I^i(w)^*, P(w)] < 0$. It remains to establish that $G_1^i(F, Y_i^{i*}) < 0$. For suppose $[P(w) - F] \geq 0$ and, that $Y_i^{i*} \geq 0$. Then, $D^i [r(w)' Z^i(w) + Y_i^{i*} [P(w) - F]] \leq D^i [r(w)' Z^i(w)]$. Hence,

$[P(w) - F] V_{11} \{r(w)' Z^i(w) + Y_i^{i*} [P(w) - F], P(w)\} \geq -[P(w) - F] D^i [r(w)' Z^i(w)]$. $V_1 \{r(w)' Z^i(w) + Y_i^{i*} [P(w) - F], P(w)\}$. If $[P(w) - F] \leq 0$, this relationship still holds. Hence,

$\sum_{w \in \Omega} g(w) [P(w) - F] V_{11} [I^i(w)^*, P(w)] \geq - \sum_{w \in \Omega} g(w) [P(w) - F] D^i [r(w)' Z^i(w)] V_1 [I^i(w)^*, P(w)]$. For members of group A, $D^i [r(w)' Z^i(w)] = D^i [S_x^i X(w)]$ is strictly monotone decreasing with respect to w . For individuals of group, $D^i [S_y^i Y, P(w)]$ is strictly monotone decreasing with respect to w . Hence, from (6),

$$\sum_{w \in \Omega} g(w) [P(w) - F] V_{11} [I^i(w)^*, P(w)] > 0 \text{ and } (D\varphi^i(F) / dF) < 0.$$

The case of $Y_i^{i*} < 0$ is similar with appropriate changes in sign.)

Following Arrow, if relative risk is constant, the willingness to accept a bet should remain unchanged as the bet and income are increased proportionately, this leads to LEMMA 5.

LEMMA 5 : “In this example, with properties in the above-mentioned polemic points, especially, with $U(\cdot, \cdot)$, let Y_i^{i*} denote an optimal choice given exogenous endowment $r(w)' Z^i(w)$, and some fixed $F \in E$. Then, if the endowment changes to $k[r(w)' Z^i(w)]$, the optimizing choice will increase to kY_i^{i*} for the same fixed F .”

(PROOF of the LEMMA 5) : (Following Stiglitz’s paper in 1969, constant relative risk averse functions are of one of the following forms where $a(\cdot)$ and $d(\cdot)$ are functions of $P(w) : V [I^i(w), P(w)] = a [P(w)] \ln [I^i(w)] + d [P(w)]$, and $V [I^i(w), P(w)] = a [P(w)] [I^i(w)]^{c+1} + d [P(w)]$. the proof follows immediately from (6).).

(COROLLARY to the LEMMA 5) :

(If $Y_i^{i*} = 0$ initially, then as $r(w)' Z^i(w)$ changes to $k[r(w)' Z^i(w)]$, Y_i^{i*} will remain zero).

From LEMMA 5, one may treat the economy as if consisting of two individuals in which A has all of (X) in all states and B has all of (Y). Each individual treats prices as parameters. As tastes are identical and homothetic, let $\delta(w)$ denote the elasticity of

substitution in consumption of each individual in state $w \in \Omega$. Also, let $R(w)$ denote the ratio of the share of A of national income in terms of (X) to the share of B. Then, $R(w) = \{X(w) / P(w) Y\}$. The findings are summarized in the following proposition.

PROPOSITION II * * * * * :

“Under the assumptions of the example with properties in the above-mentioned polemic points, especially, concerned with $U(\cdot, \cdot)$, suppose $\delta(w) = \delta^*$ for each $w \in \Omega$. (a) If $0 < \delta^* < 1$, then $Y_f^A > 0$, (b) If $\delta^* = 1$, then $Y_f^{A*} = Y_f^{B*} = 0$ and (c) If $\delta^* > 1$, then $Y_f^{A*} < 0$.”

(PROOF of the PROPOSITION II) :

(Case (a)) will be considered in detail as follows :

$G^A(F, Y_f^A) = \sum_{w \in \Omega} g(w) [P(w) - F] V_1 \{X(w) + Y_f^A [P(w) - F], P(w)\}$. By LEMMA 2 and LEMMA 3, there exists some F'' such that $G^A(F'', 0) = 0$. Define $s \in \Omega$ such that $P(s) \leq F'' < P(s+1)$. Let $G^B(F, Y_f^B) = \sum_{w \in \Omega} g(w) [P(w) - F] V_1 [P(w) Y + Y_f^B [P(w) - F], P(w)]$. By homotheticity, with $0 < \delta^* < 1$, $R(w)$ is a strictly decreasing function of w . Let $k = 1 / R(s)$. Then, $X(w) / R(w) > kX(w)$, $w = s+1, \dots, n$.

By LEMMA 1, $V_{11}(\cdot, \cdot) < 0$, and by construction $P(w) - F'' > 0$ for $w = s+1, \dots, n$.

Therefore,

$\sum_{w=s+1}^n g(w) [P(w) - F''] V_1 [X(w) / R(w), P(w)] < \sum_{w=s+1}^n g(w) [P(w) - F''] V_1 [kX(w), P(w)]$. Similarly, for $w = 1, 2, \dots, s$, $(X(w) / R(w)) \leq kX(w)$ and $P(w) - F'' \leq 0$. Therefore,

$G^B(F'', 0) < \sum_{w=1}^n g(w) [P(w) - F''] V_1 [kX(w), P(w)]$. But by the Corollary to

LEMMA 5, $G^A(F'', 0) = 0 \implies \sum_{w=1}^n g(w) [P(w) - F''] V_1 [kX(w), P(w)] = 0$.

Therefore, $G^B(F'', 0) < 0$, Therefore, $G^B(F'', 0) < 0$. By a similar argument there exists an F' such that $G^B(F', 0) = 0$ and $G^A(F', 0) > 0$. By LEMMA 1, $G^B(F'', 0) < 0 \implies \varphi^B(F'') < 0$ and $G^A(F', 0) > 0 \implies \varphi^A(F') > 0$. So, here one can define $\varphi(F) = \varphi^A(F) + \varphi^B(F)$. So, by LEMMA 2, $\varphi(F)$ is, in fact, continuous. As $\varphi(F') < 0$, $\varphi(F') > 0$, there exists an F^* such that $F' < F^* < F''$ and $\varphi(F^*) = 0$. F^* is an equilibrium forward rate with $\varphi^A(F^*) > 0$. Uniqueness follows immediately from monotonicity of LEMMA 4.

Case (b) : If $\sigma^* = 1$, then $P(w) Y / X(w) = k$ and by LEMMA 5, A and B will always be on the same side of the market. The equilibrium solution must be $Y_f^{A*} = Y_f^{B*} = 0$.

Case (c) : This case follows from the case (a) with appropriate changes in sign. Roughly speaking, if $0 < \sigma^* < 1$, the value of terms of (X) of the exogenous

endowment of B increases more as w increases than does the exogenous endowment of A.)

Concluding Remarks — Welfare Implications of Unconditional Contracts —

The purpose of this section is to analyze the welfare implications of unconditional contracts. It was shown by Arrow's paper in 1971 that complete contingent commodity markets are sufficient but not necessary for Pareto optimal allocations. And with securities yielding one dollar in each state, participants would face the same market opportunities as with complete markets. It is shown in this section that, in some circumstances, contracts conditioned on subsets of states may also suffice. The strongest result is the following.

PROPOSITION III * * * * * :

“Suppose in two states of the economic system, there exists a competitive equilibrium with complete contingent contracts and with no trade in spot markets in which the spot price of (Y) in terms of (X) differs in each of the two states. Then one equilibrium to the model with complete contingent markets and subsequent spot markets has the property that all contracts are unconditional.”

In each of the two situations described in Proposition III, the same market opportunities are available. As a matter of notation, prices and quantities of the equilibrium with contingent contracts are denoted with a superscript,*. With complete markets $I^i(w)^* = r(w)^* Z^i(w) + P(w)^* Y^i(w)^* + X^i(w)^*$, $w = 1, 2$. It will be shown that individual i can achieve the same income in terms of (X) with appropriately chosen unconditional contracts. That is, there exists a solution $\{X_i^*, Y_i^*\}$ to the following equations :

$$(8) \quad \begin{bmatrix} 1 & P(1)^* \\ 1 & P(2)^* \end{bmatrix} \begin{bmatrix} X_i^* \\ Y_i^* \end{bmatrix} = \begin{bmatrix} X^i(1)^* + P(1)^* Y^i(1)^* \\ X^i(2)^* + P(2)^* Y^i(2)^* \end{bmatrix}$$

Recall that $P(w)^* = P_y(w)^* / P_x(w)^*$, $w \in \Omega$. Initially,

$\sum_{w=1}^2 [P_x(w)^* X^i(w)^* + P_y(w)^* Y^i(w)^*] = 0$. Therefore, the elements of the matrix on the right side of (8) are equal only if both are zero. Hence, if $P(1)^* \neq P(2)^*$, (8) is consistent and has a unique solution. Then each individual has the same consumption opportunity set in each state as with contingent contracts. It remains to show that all budget constraints be satisfied, and all markets are cleared with same spot market prices. Let

$$F^* = \sum_{w=1}^2 P_y(w)^* / \sum_{w=1}^2 P_x(w)^*$$

Then, from (8),

$$(9) \quad \begin{cases} P_y(1)^* Y^1(1) + P_x(1)^* X^1(1)^* = P_y(1)^* Y_f^1 + P_x(1)^* X_f^1 \\ P_y(2)^* Y^1(2)^* + P_x(2)^* X^1(2)^* = P_y(2)^* Y_f^1 + P_x(2)^* X_f^1 \end{cases}$$

By summing equation (9),

$F^* Y_f^1 + X_f^1 = 0$ and the budget constraint of individual i for unconditional contracts is satisfied. Given equilibrium in contingent markets,

$\sum_{i=1}^m [X^i(w)^* + P(w)^* Y^i(w)^*] = 0, w \in \Omega$. By construction $P(w)^* Y^i(w)^* + X^i(w)^* = Y_f^i [P(w)^* - F^*], w \in \Omega$. Then, $\sum_{i=1}^m Y_f^i [P(w)^* - F^*] = 0$ for each $w \in \Omega$. By the hypothesis, $[P(w)^* - F^*] \neq 0$ for every $w \in \Omega$. Then, $\sum_{i=1}^m Y_f^i = 0$ and unconditional markets are in equilibrium. This leads immediately to proposition IV.

PROPOSITION IV * * * * * :

“Given the hypotheses of proposition III, the outcome in the model restricted exogenously to unconditional contracts and subsequent spot markets is Pareto optimal.”

(PROOF of the PROPOSITION IV) :

(The outcome with complete contingent markets and no active spot markets is Pareto optimal, and under the stated conditions each individual can acquire the same consumption bundles.)

The above results suggest that with more states than two, the outcome of the model restricted to spot markets and unconditional contracts is Pareto-non-optimal. To pursue this, the following LEMMA is needed.

LEMMA 6 : “Suppose in an n -states economic system there exists an equilibrium in the model restricted to unconditional contracts and subsequent spot markets. In general, for more states than two, all individuals will not have the same rates of commodity substitution across states.”

(PROOF of the LEMMA 6) : (The case $n=3$ will be considered. Let the superscript * denote prices of the initial equilibrium. Suppose the conclusion of the LEMMA 6 is false, and let R' denote the rate of commodity substitution of (X) in state one for (Y) in state two, and let R'' denote the rate of substitution of (X) in state one for (Y) in state three. Then, it is sufficient to show that in general the following equations are inconsistent. I. e.,

$$R' = \frac{g(1) V_1^1 \{r(1)^* Z^1(1) + Y_f^1 [P(1)^* - F^*], P(1)^*\}}{g(2) P(2)^* V_1^1 \{r(2)^* Z^1(2) + Y_f^1 [P(2)^* - F^*], P(2)^*\}}$$

$$R'' = \frac{g(1) V_1^1 \{r(1)^* Z^1(2) + Y_f^1 [P(1)^* - F^*], P(1)^*\}}{g(3) P(3)^* V_1^1 \{r(3)^* Z^1(3) + Y_f^1 [P(3)^* - F^*], P(3)^*\}}$$

Recall that $V_1^1 [\cdot , P(w)]$ is continuous with respect to Y_1^1 , and $V_1^1 [0, P(w)] = \infty$. Inconsistency follows from the bankruptcy constraints for unconditional contracts and the intermediate value theorem ; in general there is no Y_1^1 which solves both equations. Extensions for $n > 3$ follow similarly.)

PROPOSITION V***** :

“With more states than two, the outcome of the model restricted exogenously to unconditional contracts and subsequent spot markets is in general Pareto non-optimal. There exists in general an allocation which is Pareto superior and which could be supported, with a state-dependent income redistribution program, by a competitive equilibrium in contingent commodity markets.”

(PROOF of the PROPOSITION V)³⁾ :

(The necessary and sufficient conditions for Pareto-optimal allocations can be derived in the usual way by the maximization of the expected utility of one individual subject to fixed expected utility levels of the other $m-1$ individuals.)

By the way, on the assumption that tastes are identical and homothetic, it is possible to make clearly welfare comparisons. If tastes are identical and homothetic, then spot market prices are independent of the direction and type of forward contracts. If there are active forward markets, and if an individual chooses not to participate in such markets, then his consumption possibility set is precisely what it would have been had there been no forward market at all.

In this sense, the possibility of forward transactions can only make him better off.

So herewith one can propose the following PROPOSITION VI as the concluding remark : i. e.,

PROPOSITION VI***** :

“If tastes are identical and homothetic, the outcome in the model restricted to unconditional forward contracts with subsequent spot markets is Pareto noninferior and possibly Pareto superior to the outcome with all forward markets prohibited.”

Notes

- 1) Dreze, J. : Econometrics and Decision Theory, *Econometrica* 40 1–17 (1972)
- 2) Radner, R. : Existence of Equilibrium of Plans, Prices and Price Expectations in a Sequence of Markets, *Econometrica* 40. 289–303 (1972)
- 3) Balasko, Y. and D. Cass : The Structure of Financial Equilibrium with Exogenous yields : the case of Incomplete Markets, *Econometrica* 57, 135–163 (1989)

邦 文 要 約

先物投機型交換経済システムにかんする不完全不均衡動学の理論

この小論では、先物投機型の交換経済システムにおける各種の外生制約因子に事実上なっている各種の不完全不均衡の存在とその諸属性に関する吟味と検証を試みる。以下この小論での各論点をしめす。

1, 先物市場の役割とその社会的厚生上のメリットのチェック. 2, 先物市場の外生制約因子と投機市場取り引き均衡モデルとの関連分析. 3, 危険忌避とヘッジ取り引きとの関連分析. 4, 先物型の消費財貨購入取り引きが危険忌避のための取り引きの必要十分因子にはならないという論定にかんする吟味. 5, 社会的厚生上パレート最適をみたすスポット取り引きと先物取り引きとの対比関連分析.

(付記) : この小論は、1991年度計量経済学会国際年次大会 (於: ニューサウスウェールズ大学, オーストラリア, 7月11~14日, 筆者の招待論文報告は7月12日) での招待報告論文を全面的に書き改めたものである。