A Study on Informational Temporary Equilibrium Theories

Tokumaru KIMURA

Introduction

Generally speaking, if there are more than two possible states, or if different agents observe different data, the general equilibrium process may continue for more than two stages. The intermediate outcomes will be called informational temporary equilibria. If the number of states is finite, the process will stop in a finite number of steps, and the final outcome will be called an equilibrium.

This paper analyses the relation between equilibrium and expectations-equilibrium. The data observed by the i th agent will be assumed to be represented by a function f^i of the market data(p, y). The first result states that data structures which generally admit expectations-equilibria have the characteristic property that for each i, either f^i is constant or $f^i(p, y) \neq f^i(p', y')$ whenever$(p, y^i) \neq (p', y'^i)$. Such data structures are called admissible. This result generalizes an admissibility characterization obtained in Jordan's paper (1976) for data structures in which all agents have the same continuous data functions.A data structure is called eventually admissible if it generates equilibria which are expectations-equilibria for an admissible data structure. A characterization of eventually admissible data structures is given by the Theorem which is concerned with data structures. Any way, in this paper, we discuss the requirement that temporary equilibrium trades are not be consummated. An example is presented to demonstrate that this requirement is essential even to the existence of temporary equilibria. And the definitions of temporary equilibrium and equilibrium are extended to the general case. At any rate, suppose that the class of stochastic environments is enlarged by admitting the possibility of more than two events. If only finitely many events are admitted, the general definitions have obvious generalizations. If Σ is the set of future states, an information structure can be modelled as an N-tuple of partitions of $\Omega \times \Sigma$. A sequence of temporary equilibria would successively refine each agent's partition, and an equilibrium would be reached in a finite number of stages. However, if Σ is an infinite set, examples are easily constructed for which an equilibrium is not reached in any finite number of stages. A temporary equilibrium associates a message with each

(w, σ), so it might seem natural to define an equilibrium as the function which associates with each (w, σ) the limit of an associated sequence of temporary equilibrium messages. However, the existence of equilibrium would then depend of the existence of this limit, which not in general be an informational issue. Although the messages associated with each state may not be convergent, for each i, the information sequence $\{\eta_t^i\}_{t=0}^{\infty}$ (of either partitions of Borel Fields) increases to its least upper bound, η_*^i, consider a function which associates with each (w, σ) a competitive equilibrium message for utility functions conditioned on the information η_*^i for each i. If the resulting data do not increase any agent's information, we will call this function an equilibrium.

Method

① Static Exchange Environments : There are N agents, indexed by the superscript i, and J commodities, indexed by the subscript j, with $2 \leqslant N < \infty$ and $2 \leqslant J \leqslant \infty$. The i th agent has a consumption space X^i, an endowment space Ω^i and a space of net trades $Y^i = X^i - \Omega^i$. Let R_+^J denote the nonnegative orthant of R^J. We will assume that for each i, $\Omega^i = R_+^J \div \{0\}$ and $X^i = \mathrm{int}\ R_+^i$. Let $X^i = \Pi_{i=1}^N X^i$, $\Omega = \{w \in \Pi_{i=1}^N \Omega^i : \Pi_{i=1}^N w_j^i > 0$ for each $j\}$ and let $Y = \{y \in \Pi_{i=1}^N Y^i : \Sigma_{i=1}^N y^i = 0\}$. For each i, let u^i denote the set of continuous, strictly concave, and strictly increasing utility function $u^i : X^i \rightarrow R$, with the additional property that for each $x^i \in X^i$, the closure in R^J of the set $\{x'^i \in X^i : u^i(x'^i) \geqslant u^i(x^i)\}$ is contained in X^i. Let $U = \Pi_{i=1}^N U^i$.

The space of static exchange environments, E, is defined by $E = \Omega \cdot U$. Defining $E^i = \Omega^i \times U^i$, and making the obvious identifications, we have $E = \Pi_{i=1}^N E^i$. A generic element of E is denoted e, with the identifications $e = (w, u)$and $e = (e^1, \ldots\ldots, e^N)$. The definition of U^i insures that equilibrium allocations will be interior, which facilitates the use of calculus. It will be explained that the results in this paper would be unaffected if we redefined X^i to be R_+^J, and U^i to be the set of continuous, strictly concave, and strictly increasing functions of R_+^J.

A two-event stochastic environment associates static environments e_a and e_b with the respective states. A stochastic environment has the following interpretation. In state a, an endowment $w_a = (w_a^1, \ldots\ldots, w_a^N)$ is realized, and trading ensues. An allocation $x_a = (x_a^1, \ldots\ldots, x_a^N)$ is determined that utilities $u_a^i(x_a^i)$ are realized. The process for state b is exactly analogous. Some agents, including those for whom $w_a^i \neq w_b^i$, will initially recognize which state has occured. These agents will trade to maximize u_a^i and u_b^i in the respective states. Other agents may be unable to discern the state initially, and in the absence of further information, will trade to maximize their expected utility $\lambda^i u_a + (1-\lambda) u^i_b$ in each state, where λ is the probability of state a. Thus a stochastic environment is described by the two states and their probabilities, and the initial distribution of information. This definition is stated formally below.

② Stochastic Exchange Environments : An information structure is an N-tuple of numbers $\eta = (\eta^1, \ldots\ldots \eta^N)$ where $\eta^i \in \{0,1\}$ for each i. The ith agent is said to be informed if $\eta^i = 1$, and in uninformed by $S=\{s=(\eta, \lambda, e_a, e_b); e_a, e_b \in E,\ 0<\lambda<1$, and η is an information structure such that for each i, $\eta^i = 1$ if $w^i_a \neq w^i_B\}$. The number λ is interpreted as the probability of state a. The realization function $r_a : S \to E$ and $r_b : S \to E$ are defined coordinatewise by :

$$r_a^i(s) = \begin{cases} (w^i_a, u^i_a) \text{ if } \eta^i = 1 \text{ ; and} \\ (w^i_a, \lambda u^i_a + (1-\lambda) u^i_b) \text{ ; if } \eta^i = 0, \end{cases}$$

$$r^i_b(s) = \begin{cases} (w^i_b, u^i_b) \text{ if } \eta^i = 1 \text{ ; and} \\ (w^i_b, \lambda u^i_b + (1-\lambda) u^i_b) \text{ ; if } \eta^i = 0, \end{cases}$$

where $s=(\eta, \lambda, e_a, e_b)$.

③ Remarks : The function r^i_a associates with each stochastic environment a static environment consisting of the endowments and expected utility functions realized in state a, given each agent's information. The static environment $r_a(s)$ can be regarded as an "Initial Realization", in contrast to the "Final Realization"(w_a, u_a). We now describe the process by which agents use market data to supplement their initial information.

④ Let $\underline{\triangle}$ denote the relative interior of the unit simplex in R^l_+, and let $M = \{(p, y) \in \underline{\triangle}\, x Y : \Sigma_i y^i = 0$ and for each i, $py^{\mathrm{I}} = 0\}$. Define the correspondence $\mu : E \to M$ by setting $\mu(e)$ equal to the set of competitive equilibrium prices and net trades for e. In the definition of the competitive allocation mechanism presented in the paper of Mount and Reiter (1974), M is the competitive message space, and μ is the competitive message process. Hence elements of M will be called messages, and denoted by either (p, y) or m. For each i, a data function is a function f^i on M to an arbitrary set. A data structure is an N-tuple $\boldsymbol{F} = (f^1, \ldots\ldots, f^N)$ of data functions.

⑤ For a stochastic environment $s = (\eta, \lambda, e_a, e_b)$ and a data structure f, a sequence of temporary equilibria is a finite sequence of information structure, $\{\eta_t\}^T_{t=0}$, and message pairs, $\{(m_{at}, m_{bt})\}^T_{t=0}$ with (i) $\eta_0 = \eta$; and for each i and each $t \geq 1$,

$$\eta_t^i = \begin{cases} 1 & \text{if } \eta_{t=0}^i = 1 \text{ or } f^i(m_{at}) \neq f^i(m_{bt}) \text{; and} \\ 0 & \text{otherwise;} \end{cases}$$

(ii) for each $t \geq 1$, $m_{at} \in \mu * r_a(\eta_{t-1}, \lambda, e_a, e_b)$; and $m_{bt} \in \mu * r_b(\eta_{t-1}, \lambda, e_a, e_b)$; and (iii) $\eta_T = \eta_{T-1}$. The pair(m_{at}, m_{bt}) is said to be an equilibrium for(s, f). The above definitions embody the requirement that the trades y_{at} and y_{bt}, $t < T$, are not consummated, and have only an informational influence on the equilibrium trades y_{aT} and y_{bT}. Since there are N agents and only two states, there will always exist a sequence of temporary equilibria with $T \leq N-1$. In stage t, the i th agent uses the observed data $f^i(m_{at})$ or $f^i(m_{bt})$ to supplement his previous information η_{t-1}^i. An implication of (i) is that for each t, $\eta_t \leq \eta_{t+1}$, so no information of (i) is ever lost. Condition (ii) states that the messages m_{at} and m_{bt} are competitive equilibria for the respective realized static environments $r_a(\eta_{t-1}, \lambda, e_a, e_b)$ and $r_b(\eta_{t-1}, \lambda, e_a, e_b)$. Condition (iii) is the stationarity condition, which states that the pair (m_{at}, m_{bt}) does not generate additional information for any agent. In particular define the information structure $\hat{\eta}$ by

$$\hat{\eta} = \begin{cases} 1 & \text{if } \eta^i = 1 \text{ or } f^i(m_{aT}) \neq f^i(m_{bT}) \text{; and} \\ 0 & \text{otherwise.} \end{cases}$$

Then (iii) implies that $\hat{\eta} \leq \eta_{T-1} = \eta_T$. If $\hat{\eta} = \eta_{T-1}$, then equiblirium (m_{aT}, m_{bT}) would be an expectations equilibrium, as defined below.

⑥ Expectations-Equilibrium : An expectations-equilibrium for a stochastic environment is $s = (\eta, \lambda, e_a, e_b)$ and a data structure F is a pair $(m_a, m_b) \in MxM$ such that $m_a \in \mu * r_a(\hat{\eta}, \lambda, e_a, e_b)$and $m_b \in \mu * r_b(\hat{\eta}, \lambda, e_a, e_b)$, where $\hat{\eta}$ is defined by

$$\hat{\eta}^i = \begin{cases} 1 & \text{if } \eta^i = 1 \text{ or } f^i(m_a) \neq f^i(m_b) \text{;} \\ 0 & \text{otherwise.} \end{cases}$$

⑦ Notation : Given a metric space Z, $\beta(Z)$ denotes the Borel Field of subsets of A, and $M(Z)$ denotes the space of Borel probability measures on Z, endowed with the topology of weak convergence. Unless otherwise noted, all functions will be assumed to be Borel measurable functions taking values in a Borel subset of a complete separable metric space. Given functions h_1 and h_2, h_1 will be said to be h_2-measurable if there exists a function h_3 such that $h_1 = h_3 * h_2$. Given an indexed collection of functions $h_2 : Z \to Z$, $\alpha \in A$, the function V_α, h_α is the function $(h_\alpha)_{\alpha \in A} : Z \to \Pi_{\alpha \in A} Z$. If $\varphi \in M(Z)$, and $P(\cdot)$is a property of elements of Z, the statement $\varphi\{z :$

$P(z)\} = 1$ will be written $P(z)[\varphi]$. If there is a function h_3 such that $h_1 = h_3 \cdot h_2[\varphi]$, h_1 will be said to be h_2 measurable$[\varphi]$.

⑧ Definitions : Let Σ be a compact metric space containing more than one element. For each i, let V^i denote the set of utility functions $v^i : X^i \times \Sigma \to R$ such that (i) v^i is continuous ; and (ii) for each $\sigma \in \Sigma$, $v^i(\cdot, \sigma) \in U^i$. Let $V = \Pi_{i=1}^N V^i$. An information structure is an N-tuple $\eta = (\eta^1, \ldots\ldots, \eta^N)$ of functions on $\Omega \times \Sigma$ with the property that for each i, the projection $(w, \sigma) \to w^i$ is η^i-measurable. A stochastic environment s consists of an initial information structure η, a probability measure $\varphi \in M(\Omega \times \Sigma)$, and an N-tuple of utility functions $v \in V$. The set of stochastic environments is again denoted S. A data is an N-tuple $F = (f^1, \ldots\ldots, f^N)$ of functions on M. A sequence of temporary equilibria for a stochastic environment is $s = (\eta, \varphi, v)$ and a data structure F is a sequence of information structure η_t, $t \geqq 0$, and functions $g_t : \Omega \times \Sigma \to M$, $T \geqq 1$, such that (i) $\eta_0 = \eta$; and for each i and each $t \geqq 1$, $\eta_t^i = \eta_{t-1}^i V(f^i, g_t)$; and(ii)for each $t \geqq 1$, $g(w, \sigma) \in \mu([w^i, E\{v^i | \eta_{t-1}^i(w, \sigma)\}]_{i+1}^N)[\varphi]$, where $E\{v^i | \eta^i(w, \sigma)\}$ denotes the utility function $x^i \to E\{v^i(x^i, \cdot) | \eta^i(w, \sigma)\}$. For each i let $\eta_*^i = V_{t=0}^{\infty} \eta_t^i$. Suppose there is a function $g_* : \Omega \times \Sigma \to N$ such that (iii) $g_*(w, \sigma) \in \mu([w^i, E\{v^i | \eta_*^i(w, \sigma)\}]_{i=1}^N)[\varphi]$, and (iv) for each $i, f^i. g_*$ is $\eta_*{}^i$-measurable$[\varphi]$. Then g_* is an equilibrium for (s, F). An expectations-equilibrium for (s, F) is a function $g : \Omega \times \Sigma \to M$ such that $g(w, \sigma) \in \mu([w^i, E\{v^i | \eta^i(w, \sigma)\}]_{i=1}^N[\varphi]$, where for each i, $\hat{\eta}^i = \eta V(f^i, g)$.

Examinations and Verifications

① The compactness of Σ, together with the continuity of the utility functions v^i, insures the existence of expected utility. So, the above-mentioned condition (iv) is the informational stationarity condition.

② A two events stochastic environment $(\eta_0, \lambda, e_a, e_b)$ can of course be identified with a stochastic environment (η, φ, v) by choosing $\sigma_a \neq \sigma_b \in \Sigma$, and defining (i) $\varphi(\{(w_a, \sigma_a)\}) = \lambda$ and $\varphi(\{(w_b, \sigma_b)\}) = 1 - \lambda$; (ii) let $r = d(\sigma_a, \sigma_b)$, where d is the metric on Σ, and for each i, define $v^i : X^i \times \Sigma \to R$ by $v^i(x^i, \sigma) = [a(\sigma, \sigma_b)/r] \cdot u^i{}_a(x^i) + [d(\sigma, \sigma_a)/r] \cdot u^i{}_b(x^i)$ for each $(x^i, \sigma) \in X^i \times \Sigma$; and (iii) for each i, let η^i be trivial if $\eta_0{}^i = 0$, and let η^1 be the identity on $\Omega \times \Sigma$ if $\eta_0{}^i = 1$. Also, it is easily checked that (m_a, m_b) is an equilibrium for $(\eta_0, \lambda, e_a, e_b)$, if and only if (η, φ, v) has an equilibrium g_*, with $g_*(w_a, {}_a) = m_a$ and $g_*(w_b, {}_b) = m_b$. The exactly analogous statement holds for expectations-equilibria.

③ A data structure F is admissible if for each stochastic environment s, there exists an expectations-equilibrium for (s, F). If there exists a data structure F' such that for each $s \in S$, every equilibrium for (s, F') is an expectations-equilibrium for (s, F), then F is eventually admissible. So the advantage of the above definition in the many-event case is that it does not require the existence of equilibrium.

④ Let h_1 and h_2 be functions on $\Omega\times\Sigma$ and let $\varphi \in M(\Omega\times\Sigma)$. Suppose that for each (w, σ), $(w', \sigma') \in \Omega\times\Sigma$, $h_2(w, \sigma)=h_2(w', \sigma')$ implies $h_1(w, \sigma)=h_1(w', \sigma')$. Then h_1 is h_2 measurable$[\varphi]$.

Proof : Let K be a compact subset of $\Omega\times\Sigma$ such that h_1 and h_2 are continuous on K. Then there is a continuous function c on $h_1(K)$ such that $h_2|K=c\cdot h_1|K$. By Lusin's Theorem (in Halmos' paper (1950), p. 244), there is a Borel set $c \in \Omega\times\Sigma$ such that $\varphi(C)=1$ and C is a countable union of compact sets on which h_1 and h_2 are continuous. Therefore, $h_1(C)$ is a Borel set, and there is a Borel measurable function h_3 on $h_1(C)$ such that $h_3|C=h_3\cdot h_2|C$. This completes the proof.

⑤ THEOREM : (A) : A data structure F is admissible if and only if for each i, either (i) f^i is trivial ; or (ii) $f^i(p, y)\neq f^i(p', y')$ whenever $(p, y^i)\neq(p', y'^i)$. (B) : A data structure F is eventually admissible if and only if for each i, either (i) f^i is trivial for (ii) $f^i(p, y)\neq f^i(p', y')$ whenever $p=/p'$ and the weak axiom is satisfied for agent i. (C) : If F is eventually admissible, let F^* be a data structure such that for each i, (i) if f^i is trivial then f^{*i} is trivial ; and (ii) if f^i is not trivial then f^{*i} is the projection $(p, y)\rightarrow(p, y^i)$. Then for each stochastic environment s, every equilibrium for (s, F)is an expectations-equilibrium for (s, F^*).

Proof : For (A), it only remains to prove sufficiency. Let F satisfy (A, (ii)) or (A, (i)) for each i, and let $s=(\eta, \varphi, v)\in S$. Let η' be the information structure such that for each i, $\eta'^i=\eta^i$ if f^i is trivial, and η'^i is the identity on $\Omega\times\Sigma$ if f^i is not trivial. Let $g : \Omega\times\Sigma\rightarrow M$ be a Borel measurable selection from the correspondence $(w, \sigma)\rightarrow\mu([w^i, E\{v^i|\eta'^i(w, \sigma)\}]_{i=1}^N)$. For each i, let $\hat{\eta}^i=\eta^i V(f^i, g)$. If f^i is trivial, $\hat{\eta}^i=\eta'^i=\eta^i$, so to show that g is an expectations-equilibrium for (s, F), is suffices to show that for a.e. (w, σ), if $(p, y)=g(w, \sigma)$ then y^i maximizes $E\{v^i(w^i+y'^i)|\hat{\eta}^i(w, \sigma)\}$ subject to $py'\leq$ o for each i such that f^i is not trivial. For any i such that f^i is not trivial, let $\Pi^i : M\rightarrow\Delta x : Y^i$ be the projection and let $g^i=\Pi^i\cdot g$. By the above-stated Theorem, g^i is $\hat{\eta}^i$ measurable$[\varphi]$. For a.e. (w, σ), y^i maximizes $E\{v^i(w^i+y'^i,\cdot)|\eta'^i(w, \sigma)\}$ subject to $py'^i\leq 0$, where $(p, y^i)=g^i(w, \sigma)$. Since g^i is $\hat{\eta}^i$ measurable $[\varphi]$, it follows that y^i maximizes $E\{v^i(w^i+y^i,\cdot)|\hat{\eta}^i(w,)\}$ subject to $py'^i\leq 0$ $[\varphi]$, which proves sufficiency in (A). For (B) also, only sufficiency remains to be proved. We will establish (B) and (C) by showing that if F satisfies (B (i)) or (B (ii)) for each i, and $s=(\eta, \varphi, v) \in S$, then every equilibrium for (s, F) is an expectations-equilibrium for (s, F^*), where F^* satisfies (C (i)) and (C (ii)) for each i. Let g_* be an equilibrium for (s, F), and let $g^i_* = \Pi^i \cdot g_* = f^{*i} \cdot g_*$ for each i. We first show that for each i such that f^i is not trivial, g^i_* is η^i_* measurable $[\varphi]$, where η^i_* is defined in the above-stated definitions. Let (w, σ), (w', σ') $\in \Omega \times \Sigma$ with $\eta^i_* (w, \sigma) = \eta^i_* (w', \sigma')$, and let $(p, y^i) = g^i_* (w, \sigma)$ and $(p', y'^i) = g^i_* (w, \sigma)$, for any i such that f^i is not trivial. Then $E\{v^i | \eta^i_* (w, \sigma)\} = E\{v^i | \eta^i_* (w', \sigma')$ and $w^i = w'^i$ so (p, y^i) and (p', y'^i) satisfy the weak axiom for i. Also, since the common expected utility function is strictly concave, $y^i \neq y'^i$ only if $p \neq p'$. However, (B) implies that $p = p'$, so $g^i_* (w, \sigma) = g^i_* (w', \sigma')$. Therefore, the Theorem in ⑤ implies that g^i_* is η^i_* measurable $[\varphi]$. Repeating the argument in the first paragraph above, with η^i_* in place of η'^i, completes the proof.

Concluding Remarks

① We now consider the existence of equilibrium. First, it is necessary to assume that data functions are continuous to ensure that the equally $f^i(m) = f^i(m')$ is a closed condition. Otherwise, even if g_t converged pointwise everywhere to g_*, $f^i \cdot g_*$ might distinguish events not distinguished by $f^i \cdot g_t$ for any t. If the support of φ is countable, the sequence $\{g_t\}$ will have a pointwise a.e. convergent subsequence. The first paragraph of the proof in the below mentioned, combined with the continuity of data functions, shows that the pointwise a.e., limit of any subsequence of $\{g_t\}$ is an equilibrium. For the general case, the following result establishes the existence of equilibria for continuous eventually admissible data structures.

② Proposition : Let F be an eventually admissible data structure such that f^i is continuous for each i. Then for each stochastic environment s, (s, F) has an equilibrium.

<u>Proof</u> : Let $s = (\eta, \varphi, v)$ be a stochastic environment, and let $\{\eta_{t-1}, g_t\}_{t=1}^{\infty}$ be a sequence of temporary equilibria for (s, F). For any i and any $x^i \in x^i$, let $\{x_t^i\}_{t=1}^{\infty}$ be a sequence in X^i converging to x^i. For each i, define the function $v_t^i : \Omega \times \Sigma \rightarrow R$ by $v_t^i = E\{v^i(x_t^i, \cdot) | \eta_t^i\}$, and let $v^i_* = E\{v^i(x^i, \cdot) | \eta^i_*\}$. We will show that the sequence $\{v_t^i\}$ contains a subsequence converging to v^i_* pointwise a.e.$[\varphi]$. It suffices to show that $\{v_t^i\}$ converges in measure to v^i_* (in Halmos' paper (1950) Theorem D, p. 93). Let $\tilde{\eta}_*^i$ and $\tilde{\eta}_t^i$ denote the subfields of $\beta (\Omega \times \Sigma)$ generated by $\tilde{\eta}_t^i$, for each t, and $\tilde{\eta}_*^i$ respectively. For any $\gamma > 0$, let $A \in \tilde{\eta}_*^i$ such that (*) $\varphi(A) > 0$, and for some number c, $|V^i_* (w, \sigma) - c| \leq \gamma$ for all $(w, \sigma) \in A$. Let $\delta = \varphi(A)$, and let $k = 2 \sup\{|v^i(x_t^i, \sigma)|, t \geq 1, \sigma \in \Sigma\}$. Since $\eta^i_* = V_t \eta_t^i$, there is some t_0 and some $B \in \tilde{\eta}_{t_0}^i$ with $\varphi(A \Delta B) < a$, where $a = (\gamma^3 / 2k^2)$,

and Δ denotes symmetric difference (in Halmos' paper (1950), Theorem D, p. 56). Since $\tilde{\eta}_t^i \in \tilde{\eta}_{t+1}^i$ for each t, $B \in \tilde{\eta}_t^i$ for all $t \geqq t^0$. Let $t' > t^0$ such that for each $t > t'$, $|v^i(x_t^i, \sigma) - v^i(x^i, \sigma)| < b$ for each $\sigma \in \Sigma$, where $b = ka/(\delta + a)$. For any $t > t'$, let $C = \{(w, \sigma) \in B : v_t^i(e, \sigma) - c > 2\gamma$. By the definitions of v_t^i and v_*^i, $\int_c [v_t^i(\cdot) - v_*^i(\cdot)] d\varphi < b\varphi(c) \leqq b(\delta + a)$. Since $\int_c [v_t^i(\cdot) - v_*^i(\cdot)] \cdot d\varphi = \int_{c \cap A} [\ \ ] d\varphi + \int_{C/A} [\ \ ] d\varphi \geqq \alpha\varphi(c \cap A) - ka$, $\varphi(C \cap A) < (1/\gamma)[b(\delta + a) + ka = \gamma^2/k$.

By partitioning the interval $[-k, k]$ into k/γ subintervals of length 2γ, and applying$(v_*^i)^{-1}$, one obtains a collection of at most k/γ subsets of $\Omega \times \Sigma$ which satisfy (*) and whose union has probability one. Therefore, we have shown that for each $t > t'$, $\varphi(\{(w, \sigma) : v_t^i(w, \sigma) - v_*^i(w, \sigma) > 3\gamma\}) < \gamma$. Similarly, $\varphi(\{(w, \sigma) : v_t^i(w, \sigma) - v_*^i(w, \sigma) < -3\gamma\}) < \gamma$, which proves that $\{v_t^i\}$ converges in measure to v_*^i. Thus any subsequence of $\{v_t^i\}$ contain a pointwise a. e. convergent subsequence.

For each t, let $p_t : \Omega \times \Sigma \to \Delta$ be the function obtained by composing g_t and the projection of M into Δ. Let $p_* : \Omega \times \Sigma \to \Delta$ be a $V_{t=1}^{\infty} p_t$ meausrable selection from the correspondence $(w, \sigma) \to \cap_{t0+1} \mathrm{cl}\{p_{t0}(w, \sigma), p_{t0+1}(w, \sigma), \ldots\ldots\}$, where cl denotes closure. Define the function $g_* : \Omega \times \Sigma \to M$ by $g_*(w, \sigma) = (p, y)$, where $p = p_*(w, \sigma)$, for each $1 \leqq i < N$, y^i is the excess demand of at p determined by w^i and $E\{v^i | \eta_*^i(w, \sigma)\}$, and $y^N = -\Sigma_{i<N} y^i$, for each $(w, \sigma) \in \Omega \times \Sigma$. The result of the first paragraph above implies that $g_*(w, \sigma) \in \mu([w^i, E\{v^i | \eta_*^i(w, \sigma)\}]_{i=1}^N [\varphi]$. We will use to show that $f^i \cdot g_*$ is η_*^i measurable for each i. For any i such that f^i is non-trivial, let (w, σ), $(w', \sigma') \in \Omega \times \Sigma$ such that $\eta_*^i(w, \sigma) = \eta_*^i(w', \sigma')$. Let $(p, y) = g_*(w, \sigma)$ and $(p', y') = g_*(w', \sigma')$. By $V_{t=1}^{\infty} p_t(w, \sigma) = V_{t=1}^{\infty} p_t(w', \sigma')$, so the definition of g_* implies that $p = p'$. Let $\{t_k\}$ be an increasing sequence of integers such that $\lim_{k\to\infty} {}_{t_k}(w, \sigma) = p$. Then since $V_{t=1}^{\infty} p_t(w, \sigma) = V_{t=1}^{\infty} p_t(w', \sigma')$, the result of the first paragraph implies that $\lim_{k\to\infty} {}_{t_k}(w, \sigma) = (p, y)$ and $\lim_{k\to\infty} {}_{t_k}(w, \sigma) = (p', y')$.

Since f^i is continuous and $f^i \cdot g_t(w, \sigma) = f^1 \cdot g_t(w', \sigma')$for all t, $f^i(p, y) = f^i(p', y')$, and result follows from the above-mentioned Theorem.

References

1) Allen, B. : A Class of Monotonic Economies in which rational expectations-equilibria exists but prices do not reveal all information, *Econ. Letters*, **7**, 227〜232 (1981)

2) Green, J. : Information, Efficiency and Equilibrium, *Harvard Institute of Econ. Res.*, Discussion Paper, **284** (1973)

3) Green, J., and J. J. Laffont : Limited Communication and Incentive Compatibility, *J. Econ. Res.*, **8**, 98〜107 (1985)

4) Grossman, S. J. : An Introduction to the Theory of Rational Expectations under Asymmetric Information, *Rev. Econ. Stud.*, **48**, 541-560 (1981)

5) Grossman, S. J., and J. Stiglitz : On the Impossibility of Informationally Efficient Markets, *Amer. Econ. Rev.*, **70**, 393～408 (1980)

6) Guesnerie, R. : On Taxation and Incentives Further Reflections on the Limits to Redistribution, *EHESS Mimeo*, **18**, 20～56 (1981)

7) Hellwig, M. : On the Aggregation of Information in Competitive Markets, *J. Econ. Theory*, **22**, 477～498 (1980)

8) ——: Rational Expectations-Equilibrium with conditioning on Past Prices : A Mean Variance Example, *J. Econ. Theory*, **26**, 279～312 (1982)

9) Hirschleifer, J. : The Private and Social Values of Information and the Reward to Incentive Activity, *Amer. Econ. Rev.*, **61**, 561～574 (1971)

10) Holmstrom, B. and R. Myerson : Efficient and Durable Decision Rules with Incomplete Information, *Econometrica*, **51**, 1799～1820 (1983)

11) Jordan, J., and R. Radner : Rational Expectations in Micro Economic Models : An Overview, *J. Econ. Theory*, **26**, 201～223 (1982)

12) Kihlstrom, R. : A Bayesian Model of Demand for Information about Product Quality, *Int. Econ. Rev.*, **15**, 99～118 (1974)

13) Laffont, J. J., and E. Maskin : A Differential Approach to Dominant Strategy Mechanism, *Econometrica*, **48**, 1507～1520 (1980)

14) Lucas, R. E. : Expectations and Neutrality of Money, *J. Econ. Theory*, **4**, 103～124 (1972)

15) Milgrom, P., and N. Stockey : Information, Trade, and Common Knowledge, *J. Econ. Theory*, **26**, 17～27 (1982)

要　　旨

「情報期待均衡理論の一考察」

この小論で論証する各論点は次の通り．(1)主体 i の観測データが市場データの構造(p, y)の関数f^i でクリアできること．(2)期待均衡での許容可能なデータ構造は，f^iがコンスタントか，$f^i(p, y) \neq f^i(p', y')$のときにクリアできること．ただし，$(p, y^i) \neq (p', y'^i)$．(3) (2)のデータ構造は事実上許容可能だと判定できること．(4) J. Jordan の1976年の論文での許容可能な条件の特性化要件を(3)の判定は一般化できること．

(5) Jordan の命題は全データ構造に関して i 一般が同じ連続型のデータ関数を規定できることを指摘していること．(6) 一時的均衡取引不成立の条件が，即，この均衡の存在要件になること．(7) 一時的均衡と均衡一般との定義はともに一般均衡分析の作業でも採用可能なこと．(8) データ構造は許容可能なデータ構造をもつ期待均衡となる均衡一般が現実に存在するとき，はじめて真に許容可能になること．(9) 真に許容可能なデータ構造の特性一般に関する定理を

たてること.

(付記：この小論は1986年度国際計量経済学会研究報告年次大会(於：メルボルン大学，8/28～8/30)で筆者が研究発表した論文(発表日時・8/29，pm 2：00～3：00)を修正加筆したものである．)