

On Shadow Pricing, Information and Stability in the Open Economic Systems

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Introductory Remarks

Given the existence of distortions in market prices which cannot, for the time being, be corrected, the question arises whether it is possible to increase welfare by basing production decisions in the system-processes of public sector on some alternative set of prices. This is the essential question of shadow pricing. Since market distortions are assumed to exist, and are to be taken as given, shadow pricing is essentially a problem in the economics of the second-best. The above question can be thought of as consisting, first, of whether welfare-increasing shadow prices exist, and second, of whether it is possible to find such prices in practice. Given the usual convexity assumptions there is little controversy over the first question but there is substantial disagreement over the second.

The literature on shadow pricing falls largely into two categories. Writers in the first group emphasize the informational problems of finding the correct second-best shadow prices, although the precise nature of the informational problems involved is not usually made explicit. A familiar proposition, however, is that when there are non-traded commodities the informational problems of finding the dual solution to the second-best optimal public sector production program (shadow prices) are equivalent to the informational problems of finding the corresponding primal solution¹⁾. Thus it is suggested that unless the planner concerned with determining shadow prices is endowed with information on such crucial matters as the public sector's production possibility set, he could not possibly hope to determine the second-best optimal set of shadow prices. These writers recommend that, in practice, public sector production decisions be based on domestic market prices, both for traded and non-traded commodities, even though these prices are known to be distorted²⁾.

Writers in the second group attempt to derive rules for public sector shadow pricing from the first-order conditions for an optimization model. Sometimes this exercise is conducted relatively formally, some times rare casually, but the most common recommendation to emerge is that traded commodities should be valued at their international prices, irrespective of the existence of tariffs, and that non-traded commodities should be valued at their "foreign exchange equivalent"³⁾. There has been considerable ambiguity over the precise meaning of the latter concept, but a simplistic interpretation is that it is the rate of transformation between the non-traded commodity concerned and a traded commodity in the private sector⁴⁾. In general, writers in this category do not consider explicitly how the economy moves from some initial non-optimal position to the secondbest optimum described by the first-order conditions, although the shadow pricing rules are clearly meant to facilitate the transition. Some writers have suggested that an iterative adjustment mechanism is entailed, but its precise nature is not made explicit⁵⁾.

This has tended to obfuscate the informational and stability issues involved in such adjustment processes.

This paper attempts to clarify some of the informational and stability problems involved in shadow pricing in the context of the open economic system model. This model is described in it. And in it, also a fixed distortionary tariff is introduced and necessary conditions for a second-best optimum are derived. So, we then can interpret the meaning of these first-order conditions as the terms of shadow prices. And this paper examines the informational problems involved in moving from an initial non-optimal position to this second-best optimum by means of shadow pricing.

Method

Consider an economic system with three commodities :

I. a traded, domestically produced consumption good (e. g. : wheat), whose fixed international price is normalized at unity ;

II. a traded input, not domestically produced (e. g. : fertilizer), whose fixed international price is normalized at unity ; and

III. a non-traded input (e. g. : land), whose fixed total supply is normalized at unity.

There is a single consumer whose utility function is simply $U=c_1$, where c_1 is his consumption of commodity I ; $c_1 \geq 0$. There are two producers, a private firm and a public firm. The private firm's production function is given by $y_1=f(y_{II}, y_{III})$, where y_1 is the firm's output of commodity I, y_{II} and y_{III} are its inputs of commodities II and III ; $y_1, y_{II}, y_{III} \geq 0$. The public firm's production function is $x_1=g(x_{II}, x_{III})$, where $x_1, x_{II}, x_{III} \geq 0$ and are defined similarly. The functions f and g are assumed to be strictly concave and twice continuously differentiable, but not necessarily identical. Commodity I is used as numeraire, so both market and shadow prices of commodities II and III are expressed in terms of commodity I. The market prices are denoted by p_{II} and p_{III} , and the shadow prices are denoted by s_{II} and s_{III} .

Commodity I can be exported to purchase commodity II, but such transactions must satisfy the trade balance constraint :

$$1. \quad g(x_{II}, x_{III}) + f(y_{II}, y_{III}) - x_{II} - y_{II} - c_1 \geq 0.$$

Furthermore, total use of commodity III must not exceed the available supply.

Thus,

$$2. \quad (1 - x_{III} - y_{III}) \geq 0.$$

There are three agents in the economy whose knowledge and behaviour concern us : the private firm's manager, the public firm's manager and a planner. The private firm's manager is assumed to know the precise form of f and the market prices p_{II} and p_{III} obtaining at any instant, but nothing else. He is assumed to behave so as to maximize profits treating market prices parametrically. His problem is thus

$$M \quad A \quad X \quad \Pi^y = f(y_{II}, y_{III}) - (p_{II}y_{II}) - (p_{III}y_{III}).$$

So, in this case, the necessary conditions are

$$f_{II} \leq p_{II}, \quad y_{II}(f_{II} - p_{II}) = 0$$

and

$$f_{III} \leq p_{III}, \quad y_{III}(f_{III} - p_{III}) = 0.$$

Similarly, the public firm's manager is assumed to know the precise form of g and the shadow prices s_{II} and s_{III} given him by the planner at any instant, but nothing else. He maximizes shadow profit, treating shadow prices parametrically. His problem is thus

$$\begin{matrix} \text{M} & \text{A} & \text{X} & \text{II}^x = g(x_{II}, x_{III}) - (s_{II}x_{II}) - (s_{III}x_{III}). \\ & & & x_{II}, x_{III} \end{matrix}$$

The necessary conditions are

$$\begin{aligned} (g_{II} - s_{II}) &\leq 0 & x_{II}(g_{II} - s_{II}) &= 0, \text{ and} \\ (g_{III} - s_{III}) &\leq 0 & x_{III}(g_{III} - s_{III}) &= 0. \end{aligned}$$

The planner is assumed to know the domestic market prices p_{II} and p_{III} obtaining at any instant, the international price of commodity II (normalized here at unity) and the magnitude of any tariff on commodity II. He is also capable of observing the private firm's input and output levels at any instant. However, unless otherwise stated, the planner is assumed not to know any details of the functions f and g , except that they are both concave. Furthermore, he is assumed not to know the magnitude of the constraint on the availability of commodity III (here normalized at unity). The planner is assumed to control only the shadow prices s_{II} and s_{III} , which he transmits to the public project. In particular, the planner has no control over the government and the planner must take them as given.

Finally, we consider the determination of p_{III} . The strict concavity of f implies, ignoring some irregular cases⁶⁾, that the Hessian matrix $[f_{ij}]_{i,j=2,3}$ of second derivatives of f is negative definite. This implies, by the implicit function theorem, that the first-order conditions for the private firm generate differentiable demand functions of the form

$$y_{II} = Y_{II}(p_{II}, p_{III}), \quad y_{III} = Y_{III}(p_{II}, p_{III}).$$

Given x_{III} and p_{II} predetermined at x_{III}^0 and p_{II}^0 , respectively, p_{III} is assumed to adjust instantaneously such that

$$Y_{III}(p_{II}^0, p_{III}) = 1 - x_{III}^0.$$

Verifications

* Necessary Conditions for the General Optimum Balances

Here, we can derive the necessary conditions for optimal public sector production in the presence of a fixed market distortion and interpret their meaning in terms of shadow prices. Consider first the "first best" optimization polemic point.

POINT 1: MAX c_I subject to 1 and 2.

In this case, we can form the Lagrangian

$$3. \quad L^* \equiv c_I + \lambda_I \{g(x_{II}, x_{III}) + f(y_{II}, y_{III}) - c_I - x_{II} - y_{II}\} + \lambda_{II}(1 - x_{III} - y_{III}).$$

The necessary conditions for optimal private sector production then are

$$\begin{aligned} f_{II}^* &\leq 1, \quad y_{II}^*(f_{II}^* - 1) = 0 \quad \text{and} \\ f_{III}^* &\leq \lambda_{II}^*, \quad y_{III}^*(f_{III}^* - \lambda_{II}^*) = 0, \end{aligned}$$

where (*) denotes evaluation of the quantity concerned at this "first best" optimum.

Assuming $y_{II}^* > 0$ the necessary conditions for optimal public sector production become

$$g_{II}^* \leq 1, \quad x_{II}^*(g_{II}^* - 1) = 0, \quad \text{and} \\ g_{III}^* \leq f_{III}^*, \quad x_{III}^*(g_{III}^* - f_{III}^*) = 0.$$

Hence, comparing these expressions with the above conditions for profit and shadow profit maximization, the “first-best” optimal shadow prices are

$$s_{II}^* = p_{II}^* = 1 \quad \text{and} \quad s_{III}^* = p_{III}^*.$$

Both the shadow price and the market price of the traded input should be its international price, while the non-traded input should be valued identically in the two sectors ($s_{III}^* = p_{III}^*$).

We now introduce a distortion into the market for the traded input in the form of a fixed tariff, t , where $-1 < t < \infty$. Assuming $y_{II} > 0$, profit maximization of the private firm now requires

$$4. \quad f_{II} = 1 + t,$$

which violates one of the necessary conditions for a “first-best” optimum. The “second-best” optimization polemic point now is as follows.

POINT 2: MAX c_I subject to 1, 2 and 4.

So, we can form the Lagrangian

$$5. \quad L^{**} \equiv c_I + \lambda_I \{g(x_{II}, x_{III}) + f(y_{II}, y_{III}) - c_I - x_{II} - y_{II}\} \\ + \lambda_{II}(1 - x_{III} - y_{III}) + \lambda_{III} \{f_{II} - (1 + t)\}.$$

Assuming $(y_{II}^{**}, y_{III}^{**}) > 0$, the necessary conditions for optimal public sector production become (assuming $f_{II}^{**}, f_{III}^{**} \neq 0$)

$$g_{II}^{**} \leq 1, \quad x_{II}^{**}(g_{II}^{**} - 1) = 0 \quad \text{and} \\ g_{III}^{**} \leq p_{III}^{**} - (t f_{II}^{**} / f_{II}^{**}), \quad x_{III}^{**}(p_{III}^{**} - t f_{II}^{**}) = 0,$$

where $(**)$ denotes evaluation at this “second-best” optimum.

The “second-best” optimal shadow prices are thus

$$6. \quad s_{II}^{**} = 1 \quad \text{and}$$

$$7. \quad s_{III}^{**} = p_{III}^{**} - (t f_{II}^{**} / f_{II}^{**}), \quad (f_{II}^{**} \neq 0).$$

Note that the presence of a fixed tariff on the traded input implies the the shadow prices of both inputs should differ from their domestic market prices—provided, in the case of the non-traded input, that $f_{II}^{**} \neq 0$. The second-best shadow price of the traded input still is international price, regardless of the existence of a tariff, while the second-best shadow price of the non-traded input depends both on the tariff and the second derivatives of the private firm’s production function. We now can consider whether this latter shadow price can be interpreted as a “foreign exchange equivalent”.

The net contribution of the private firm to aggregate foreign exchange earnings is clearly $y_I - y_{II}$. The net effect of an extra unit of commodity III on the firm’s contribution to foreign exchange earnings is therefore $(dy_I/dy_{III}) - (dy_{II}/dy_{III})$. Now, totally differentiating the private firm’s production function,

$$8. \quad dy_I = (f_{II} dy_{II}) + (f_{III} dy_{III}), \quad \text{and}$$

$$\frac{dy_I}{dy_{III}} = f_{II} \left(\frac{dy_{II}}{dy_{III}} \right) + f_{III} = (1 + t) \frac{dy_{II}}{dy_{III}} + f_{III}.$$

Totally differentiating equation 4,

9. $(f_{II, II}dy_{II}) + (f_{II, III}dy_{III}) = 0$, and hence

$$\frac{dy_{II}}{dy_{III}} = -\frac{f_{II, III}}{f_{II, II}}.$$

From 4, 8 and 9, we now can have, at the second-best optimum,

$$10. \frac{dy_{II}^{**}}{dy_{III}^{**}} - \frac{dy_{II}^{**}}{dy_{III}^{**}} = p_{III}^{**} - t \frac{f_{II, III}^{**}}{f_{II, II}^{**}},$$

which is the same as equation 7.

Thus the shadow price of the non-traded commodity is the net effect that releasing a unit of that commodity to the private firm has on aggregate foreign exchange earnings, at the optimum⁷). There seems little point in debating whether this is what previous authors “really” meant by a “foreign exchange equivalent”, since that term has been used rather ambiguously ; the point is that the above argument also demonstrates that the logical basis for the “foreign exchange equivalent shadow pricing procedure derives from the first-order conditions from an optimization problem. Note especially that for the optimal second-best shadow price of the non-traded input to be viewed correctly as a “foreign exchange equivalent”, the latter concept should not be interpreted as simply the private firm’s rate of transformation between that input and a traded output, as a more simplistic interpretation would suggest ; this is simply p_{III} in equation 7. Of course, the formal correspondence between equation 7 and the concept of a “foreign exchange equivalent” says nothing about the informational problems of detemining s_{III}^{**} . We now turn to this question.

Concluding Remarks

We now suppose the economy to be producing initially at some point removed from the second-best optimum, meaning that equations 6 and 7 are not satisfied, and consider the planner’s informational problems in finding shadow prices that will move the economy to this optimum. Clearly, there is no problem in the case of the traded input, since we have assumed its international price to be known. Our attention will focus on the case of the non-traded input.

Consider, first, the planner’s informational requirements for finding the numerical value of s_{III}^{**} in equation 7. Clearly, this value will, in general, depend on the forms of the functions f and g . This has led some writers to suggest that, economies where not all commodities are traded, the informational requirements of finding the set of shadow prices associated with an optimal production program (i. e. the dual solution) are equivalent to those of finding the physical quantites involved in that production program (i. e. the primal solution). If this were so, there would be a logical contradiction in supposing that a planner could determine the value of s_{III}^{**} in a single step without full information about g , f and the resource constraint. But this is untrue ; very partial information may be sufficient.

Suppose, for example, that the planner knows only that f is additively separable and linear in y_{III} . Then it has the form

$$11. f(y_{II}, y_{III}) = h(y_{II}) + by_{III},$$

where $b > 0$ is an unknown constant, and the functions h and g are unknown except that g

is concave and $h'' < 0$. Then 7 reduces to 12.

$$12. s_{III}^{**} = b.$$

From the necessary conditions for profit maximization it is clear that whenever $y_{III} > 0$, $f_{II} = b = p_{III}$, the observed market price.

Thus without any a priori knowledge of b , or of the functions g or h , the planner can infer the optimal shadow prices immediately; namely, $s_{II}^{**} = 1$ and $s_{III}^{**} = p_{III}$. But he is still unable to determine the physical quantities involved in the optimal public sector production program without knowledge of g , and this is possessed only by the public firm's manager.

More generally, or course, f cannot be assumed to have this convenient form and the planner's informational requirements for determining the numerical value of s_{III}^{**} in a single step will be more severe. Let us suppose these requirements to be prohibitive, for the sake of argument, and consider the informational problems in applying the "rule" given by 7 iteratively. This is what a number of writers seem to intend. At time $t=0$, let the public sector be producing nonoptimally. Let $s_{II}^0 = 1$, but

$$13. s_{III}^0 \neq p_{III}^0 - (tf_{II, III}^0 / f_{II, II}^0),$$

where numerical superscripts denote points in time. Now let observations be made on the right hand side of 13 and s_{III}^1 be set equal to this. Thus

$$14. s_{III}^1 = p_{III}^0 - (tf_{II, III}^0 / f_{II, II}^0).$$

Now let the market price of commodity III adjust fully to this change in public production, and let the private firm adjust fully to this price change. Then let the planner revise s_{III} by again applying the "rule" given by 7, so

$$15. s_{III}^1 = p_{III}^1 - (tf_{II, III}^1 / f_{II, II}^1),$$

etc.. That is, $s_{II}^t = 1$ for all t and

$$16. s_{III}^{t+1} = p_{III}^t - (tf_{II, III}^t / f_{II, II}^t), \quad t=0, 1, 2, \dots$$

What are the planner's informational requirements in this adjustment process?

Consider the term $f_{II, III} / f_{II, II}$. Differentiating equation 4 with respect to p_{III} we have⁸⁾

$$f_{II, II} \frac{\partial y_{II}}{\partial p_{III}} + (f_{II, III}) \frac{\partial y_{III}}{\partial p_{III}} = 0.$$

Hence,

$$17. \frac{f_{II, III}}{f_{II, II}} = - \frac{\partial y_{II} / \partial p_{III}}{\partial y_{III} / \partial p_{III}} = - \frac{\xi_{II, III} y_{III}}{\xi_{III, II} y_{II}},$$

wherer $\xi_{II, III} = \frac{p_{III} \partial y_{II}}{y_{II} \partial p_{III}}$ is the price elasticity of demand for commodity II with respect to p_{III} , etc..

Equation 16 now reduces to

$$18. s_{III}^{t+1} = p_{III}^t + (t \xi_{II, III}^t y_{III}^t) / (\xi_{III, II}^t y_{II}^t), \quad t=0, 1, 2, \dots$$

From the observability of p_{III}^t , t , y_{II}^t and y_{III}^t , it follows that if the ratio of elasticities ($\xi_{II, III}^t / \xi_{III, II}^t$) were known, the above iterative process would be informationally feasible.

Alternatively, suppose that the planner knows only that f is a COBB-DOUGLAS production function with the general form

$$19. f(y_{II}, y_{III}) = (a y_{II}^\alpha y_{III}^\beta),$$

where a , α and β are unknown parameters except that $a, \alpha, \beta > 0$ and $(\alpha + \beta) < 1$. Nothing is known of g except that it is concave. Differentiating twice we obtain

$$(f_{II, III}/f_{II, II}) = \{y_{II}/((\alpha - 1)y_{III})\}.$$

Now writing

$$f_{III}/f_{II} = \frac{\beta}{\alpha} \frac{y_{II}}{y_{III}} = \frac{p_{III}}{p_{II}},$$

we can obtain

$$\frac{f_{II, III}}{f_{II, II}} = \frac{p_{III}^{\alpha}}{p_{II}(\alpha - 1)}.$$

Now using the result that $\alpha = (p_{II}y_{II})/y_I$, we can have, substituting into 16,

$$20. \quad s_{III}^{t+1} = p_{III}^t \frac{y_I^t - y_{II}^t}{y_I^t - y_{II}^t(1+t)}.$$

This expression involves only observable market variables. Hence, while the informational problems of shadow pricing are very real, no logical contradiction is involved in supposing term to be solvable for some classes of environments (namely where f is known to take some specific forms), especially where shadow price is to be conducted iteratively. This does not mean, of course, that these problems are solvable for all classes of environments; but it is false to claim that the informational requirements for shadow pricing are equivalent to the informational requirements for determination of the physical characteristics of the optimal public sector production program. It is a mistake to dismiss the possibility of finding welfare-increasing shadow pricing procedures on these grounds.

Notes

- 1) The proposition is trivially false in the case where all commodities are traded.
- 2) Examples are R. S. Weckstein, "Shadow Prices and Project Evaluation in Less-Developed Countries," (Economic Development and Cultural Change XX (April 1972), 474-494,) and A. Rudra, "Use of Shadow Prices in Project Evaluation", (Indian Economic Review VII (S. S.) (April 1972), 1-15,).
- 3) Examples are P. Dasgupta and J. E. Stiglitz, "Benefit-Cost Analysis and Trade Policies," (Journal of Political Economy LXXXII (January/February 1974, 1-33), I. M. D. Little and J. A. Mirrlees, Manual of Industrial Project Analysis in Developing Countries (Paris: Organization for Economic Cooperation and Development, 1969), and I. M. D. Little and J. A. Mirrlees, Project Appraisal and Planning for Developing Countries (New York: Basic Books, 1974).
- 4) For a critique of the Dasgupta-Stiglitz argument on this point, see P. G. Warr, "A Note on Shadow Pricing with Fixed Taxes," (Discussion Paper no. 74-52 (December 1974), Department of Economics, University of Minnesota.)
- 5) See, for example, P. Dasgupta, S. A. Marglin and A. K. Sen, Guidelines for Project Evaluation (New York: United Nations Industrial Development Organization, 1972, Ch. 17.)
- 6) There are irregular cases in which the derivatives f_{II} may vanish, even though f is strictly concave, but only at a countable number of isolated points. These points form a "negligible" subset of the domain of f .
- 7) For a contrary claim, see R. Broadway, "Benefit-Cost Shadow Pricing in Open Economies: An Alternative Approach", (J. of Political Econ. LXXXIII (Apr. 1975), 419-430)
- 8) Since equation 4 must hold for all p_{III} and, from the implicit function theorem, the demand functions

$y_{II} = Y_{II}(p_{II}, p_{III})$ and $y_{III} = Y_{III}(p_{II}, p_{III})$ are differentiable, this differentiation is legitimate.

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要 約

潜在価格体系における情報 の安定性問題をめぐって

制御不能な市場価格攪乱の存在がみとめられるとき、公共部門調整システムの展開過程での生産量決定を任意の価格体系をベースにとって作業することで社会的公正水準を増幅することがはたしてできるのかという設問がストレートな形でうまれる。これはまさに潜在価格水準決定にかかわる基本的論点である。本稿では、まず開放系の経済システムモデルのシナリオで潜在価格決定に関する情報とその安定性をめぐる若干の問題を考える。つぎにこのシステムモデルの説明を試みる。第3に、次善型の最適厚生水準を保証できる要件をクリアする。最後に潜在価格決定の作業をとおして初期時点の厚生水準値一般からの、社会的厚生水準最適値導出に関する各種の情報選択問題とそれら情報の安定性をめぐる諸問題を吟味する。

(付記：本稿は昨年8月国立オーストラリア大学での国際計量経済学会において報告した論文の第2章と第3章をまとめて全面的にかき改めたものである。)