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Development and Examination on The OPEN EQUILIBRIUM MODEL (O. E. M.) concerned with The International Economic Trade

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Introduction

The logic of the arguments in this paper can be explained in simple terms. Suppose the world was not divided into countries. Then there would have emerged an equilibrium of the integrated world economy. Call it the integrated equilibrium. Now divide the world into countries. In particular, allocate the available resources and industry specific productivity shocks to countries. Then ask the question, do there exist country structures that reproduce prices, factor rewards, and aggregate quantities of the integrated equilibrium ? The answer to this question is in the affirmative, and my conditions ensure that the set of such world structures is not negligibly small. In these world structures international trade leads to factor prices equalization. It is then shown that in the resulting trading equilibria, the specified restrictions on preferences enable predictions of the factor content of net trade flows in financial assets, and expected net trade flows in goods. So, to describe an economy the following quantities are introduced:

Symbol	Dimension	Remarks
A	$m \times n$	Input Matrix
B	$m \times n$	Output Matrix
α	1×1	Growth Factor
β	1×1	Interest Factor
w^+	$1 \times n$	Vector of Exports
w^-	$1 \times n$	Vector of Imports
y^+	$n \times 1$	Vector of Export Prices
y^-	$n \times 1$	Vector of Import Prices
y	$n \times 1$	Price Vector
z^+	$m \times 1$	Vector of Profits
z^-	$m \times 1$	Vector of Losses
x^+	$1 \times m$	Vector of Upper Bounds on Intensities
x^-	$1 \times m$	Vector of Lower Bounds on Intensities
x	$1 \times m$	Intensitive Vector

At the same time, the following quantities will be used:

$$M_\alpha := B - \alpha A, \text{ and } \qquad M_\beta := B - \beta A.$$

Let $I := \{1, \ldots, m\}$ and $J := \{1, \ldots, n\}$ be sets of

indices for the rows and columns of matrix B or A. I can call the axioms of the model. For each axiom an informal remarks is given:

(A−1) $xM_\alpha = w^+ - w^-$; $w^+, w^- \geqq o$.

Production+imports are sufficient for internal demands+exports)

(A−2) $M_\beta y = z^+ - z^-$; $z^+, z^- \geqq o$.

Value of outputs+losses of unprofitable industries=Value of inputs+profits of profitable industries) .

(A−3) $w^+y^+ = w^-y^-$.

(Balance of payment condition)

(A−4) $x^+z^+ = x^-z^-$.

(Balance of profits condition)

(A−5) $xBy > o$.

(The value of outputs is positive)

(A−6) $x^- \leqq x \leqq x^+$.

(Intensity vector is within desired bounds)

(A−7) $y^+ \leqq y \leqq y^-$.

(Price vector is bounded between the export and the import prices)

(A−8) $w^{+t}w^- = o$.

(A good is not imported and exported at the same time)

(A−9) $z^{+t}z^- = o$.

(An industry does not have positive and negative profits at the same time)

An equilibrium solution to the O. E. M. is defined as follows:

(DEFINITION 1):

[(A − 7) − tuple($\bar{x}$, $\bar{w}^+$, $\bar{w}^-$, $\bar{z}^+$, $\bar{z}^-$, $\bar{\alpha}$), having the property that (A − 1) through (A−9) are fulfilled and where $\alpha = \bar{\alpha} = \beta$, is called an equilibrium solution.]

Morgenstern and Thompson have shown the existence of an equilibrium solution under the following hypotheses ((H1) through H5))

(H1) $0 \leqq x^- \leqq x^+$.
(H2) $0 \leqq y^+ \leqq y^-$.
(H3) $x^- B y^+ > 0$.
(H4) $x^- A y^- > 0$.
(H5) $A, B \geqq 0$.

Next, a short outline of the proof of Morgenstern and Thompson is given. Two corresponding linear program (LP1) and LP2), dual to each other and parameterized by α, are considered. At the same time, I can define two more programs P1) and (P2) by adding to (LP1) (resp. (P2)) a further condition Ⅵ resp. . **XII**).

Ⅰ. (LP1): Min. $-w^+y^+ + w^-y^-$.
Ⅱ. (P1): $xM_\alpha - w^+ + w^- = o$.
Ⅲ. $-x \geqq -x^+$.
Ⅳ. $x \geqq x^-$
Ⅴ. $w^+, w^- \geqq o$.
Ⅵ. $w^{+t} \times w^- = o$.
Ⅶ. (LP2): Max $x^+z^+ + x^-z^-$.
Ⅷ. (P2): $M_\alpha y - z^+ + z^- = o$.
Ⅸ. $-y \leqq -y^+$
Ⅹ. $y \leqq y^-$.
XI. $z^+, z^- \geqq o$.
XII. $z^{+t} \times z^- = o$.

Just as there exist feasible solutions to (LP1) and (LP2) under (H1) and (H2), there must exist optimal solutions to (LP1) and (LP2). Moreover, (LP1) and (LP2) have a common optimal value. To clarify the relations between the optimal solution of (LP1) and (P1), consider the following: If $\tilde{x}$ is the x-part of an optimal solution ($\tilde{x}$, $\tilde{w}^+$, $\tilde{w}^-$) of (LP1), then $\tilde{x}$ is the x-part of an optimal solution to (P1). Every optimal solution ($\bar{x}$, $\bar{w}^+$, $\bar{w}^-$) of (P1) is an optimal solution of (LP1). The optimal values of both programs are equal. A similar result holds for (LP2) and (P2).

The advantage of (P1) (resp. (P2) compared to (LP1) (resp. (LP2)) is that every feasible (and therefore, every optimal) solution ($\bar{x}$, $\bar{w}^+$, $\bar{w}^-$) of (P1) (resp.($\bar{y}$, $\bar{z}^+$, $\bar{z}^-$) of (P2)) is completely determined by $\bar{x}$ (resp. by $\bar{y}$). For given $\bar{x}$ with $x^- \leqq \bar{x} \leqq x^+$ (resp. $\bar{y}$ with $y^+ \leqq \bar{y} \leqq y^-$), the vectors $\bar{w}^+$, $\bar{w}^-$, (resp. $\bar{z}^+$, $\bar{z}^-$) are computed as follows:

XIII. $\bar{w}_j^+ = \bar{x}M_\alpha^j$, $\bar{w}_j^- = 0$ if $\bar{x}\ M_\alpha^j \geqq o$.
XIV. $\bar{w}_j^- = -\bar{x}M_\alpha^j$, $\bar{w}_j^+ = o$ if $\bar{x}M_\alpha^j < o$.
XV. $\bar{z}_i^+ = M_\alpha^i\bar{y}$, $\bar{z}_i^- = o$ if $M_\alpha^i\bar{y} \geqq o$.
XVI. $\bar{z}_i^- = -M_\alpha^i\bar{y}$, $\bar{z}_i^+ = o$ if $M_\alpha^i\bar{y} < o$.

with M_α^i, M_α^j denoting the jth row (resp.the jth column) of M_α.

In any case, a function ψ: $R_+ \to R$, which relates to every α the common optimal value of (LP1) and (LP2) (or (P1) and (P2)) , is defined. It can be shown that ψ is continuous and nondecreasing. Moreover, from (H3) and (H4) I can obtain $\psi(o) > o$ and $\psi(\alpha) \to -\infty$ for $\alpha \to +\infty$. Therefore, there are values of α for which $\psi(\alpha) = o$. It is easy to verify, that for these α's all aximos with the exception of (A－5) can be satisfied. Finally, (H3) obviously entails that for all $x \in [x^-, x^+]$ and all $y \in [y^+, y^-]$: $x\beta y > 0$, i. e., (A－5) is satisfied, too, and therefore an equilibrium solution exists. The stringency of hypothesis (H3) is apparent.

Methods

(a) : A Game-Theoretic Approach

First of all, I reformulate (P1) (resp. (P2)) by replacing I with

I' Max $w^+y^+ - w^-y^-$ resp. Ⅶ. with
VII'. Min $x^+z^+ - x^-z^-$. Then let
XVII. $T := \{x \mid x^- \leqq x \leqq x^+\}$, $P := \{y \mid y^+ \leqq y \leqq y^-\}$

Since in I' w^+ (resp. $-w^-$) are weighted with y^+ (resp. y^-), i. e.,the lower (resp. the upper) bound of the y's, it follows from XIII. and XIV. that the maximizing problem of (P1) may be formulated as

XVIII. $\max_{x \in T} \min_{y \in P} xM_\alpha y.$

Similarly,since in VII' z^+ (resp. $-z^-$) are weighted with x^+ (resp. x^-), i. e.,the upper (resp.the lower) bound of the x's, it follows from XV. and XVI. that the minimizing problem of (P2) could be formulated as

XIX. Min Max $xM_\alpha y$.

$y \in P \; x \in T$

Obviously, every pair ($\bar{x}$, $\bar{y}$) ——$\bar{x}$ and $\bar{y}$ being the x-part and the y-part of an optimal solution to (P 1) (or (LP1)) and (P2) (or(LP2)) resp. ——is a solution to the problems **XVIII** and **XIX**. Clearly, **XVIII** and **XIX** have a common optimal value. Therefore, every pair ($\bar{x}$, $\bar{y}$) is an optimal solution of the game (M_α, T. P), M_α denoting the payoff matrix, whereas T (resp. P) are the sets of strategies of the maximizing (resp. the minimizing) player. The common optimal value of (P1) (or (LP1)) and (P2) (or (LP2)) equals the value v (M_α, T, P) of the game (M_α, T, P). This gives rise to a redefinition of an equilibrium solution of the O. E, M..

(DEFINITION 1'):

[A triple ($\bar{x}$, $\bar{y}$, $\bar{\alpha}$) is called an equilibrium solution to the O. E. M. ,if (i) ($\bar{x}$, $\bar{y}$) is a minimax solution to ($M_{\bar{\alpha}}$, T. P), (ii) v$M_{\bar{\alpha}}$, T, P) = O, and (iii) $\bar{x}B\bar{y} > O$. The quantities $\bar{w}$, $^{+}\bar{w}^{-}$, $\bar{z}^{+}$, $\bar{z}^{-}$ are determined according to **XI.** —**XIV.**].

It is well-known that a minimax solution of a two-person zero-sum game can be characterized by a saddle point. For all pairs ($\bar{x}$, $\bar{y}$) being a minimax solution to (M_α, T, P),I can have:

XX. $\forall \; y \in p: \; \bar{x}M_\alpha y \geqq v\,(M_\alpha, T, P).$

XXI. $\forall \; x \in T: \; v\,(M_\alpha, T, P) \geqq xM\bar{y}$

XXII. $v\,(M_\alpha, T, P) = \bar{x}M_\alpha\bar{y}$

From equations **XX.** through **XXII.** the following relations hold: if ($\bar{x}$, $\bar{y}$) is a pair of minimax solution to (M_α, T, P), then

XXIII. $\bar{x}M_\alpha^j > O \Longrightarrow \bar{y}_j = y^+$

XXIV. $\bar{x}M_\alpha^j < O \Longrightarrow \bar{y}_j = y_j^-$

XXV. $M_\alpha^i\bar{y} > O \Longrightarrow \bar{x}_i = x_i^+$

XXVI. $M_\alpha^i\bar{y} < O \Longrightarrow \bar{x}_i = \bar{x}_i^-$

(b) : The O. E. M. as Von Neumann Model in a Vector Space Ordered by Means of Cones

For convenience,the model is realized as a pair of linear transformations, i. e., ·A, ·B of the m-dimensional linear space X (intensities) into the n-dimensional linear space W (goods),and their dual transformations A · , B · operating from Y (prices),the dual-space of W,into Z(values) ,the dual-space of X.

$$Y \xrightarrow{\cdot A,\ \cdot B} W \qquad Z \xleftarrow[A\cdot\ ,\ B\cdot]{} Y$$

Let K_T and K_P be the cones generated by the convex sets T and P, i, e.,

XXVII. $K_T := \{x \in X \mid \exists\ (t, \lambda) \in T \times R_+ : x = \lambda \cdot t\}$
XXVIII. $K_P := \{y \in Y \mid \exists\ (p, \lambda) \in P \times R_+ : y = \lambda \cdot p\}$

Obviously, K_T and K_P are closed polyhedral cones. The dual cones of K_T and K_P are denoted as K_T^* and K_P^*:

XXIX. $K_T^* := \{z \in Z \mid xz \geqq o,\ \forall x \in K_T\}$
XXX. $K_P^* := \{w \in W \mid wy \geqq o,\ \forall y \in K_P\}$

Now by means of the cones K_T and K_T^* (resp. K_P and K_P^*) partial orders $\geqq_s$, are introduced for the pairs of dual spaces X and Z (resp. Y and W). The definition of relation $\geqq_s$ will be given here for space X only:

XXXI. $x^2 \geqq_s x^1$ iff $x^2 - x^1 \in K_T$

Recalling some familar basic properties of partial orders as defined by closed polyhedral cones, the relations $\geqq_s$ are closed,homogeneous, additive, reflexive and transitive but not necessarily anti-symmetric. Due to the concept of linear spaces ordered by means of cones. the O. E. M.can —— apart from normalizing conditions —— formally be represented in the same way as the Von Neumann Model K. M. T.version).

(THEOREM 1):

[A solution $(\overline{x}, \overline{y}, \overline{\alpha})$ of the system of linear inequalities

XXXII. $xM_\alpha \geqq_s O,\ x \geqq_s o.$
XXXIII. $M_\alpha y \leqq_s o,\ y \geqq_s o.$
XXXIV. $xBy > O.$

fulfills, up to multiplication of $\overline{x}$ and $\overline{y}$ by a positive scalar, the conditions (i) through (ii) of (DEFINITION 1') and every triple $(\hat{x}, \hat{y}, \hat{\alpha})$ fulfilling (i) through (iii) of (DEFINITION 1') is a solution of the above inequalities.]

PROOF :

{Let $(\overline{x}, \overline{y}, \overline{\alpha})$ be a solution to **XXXII** through **XXXIV**. Clearly, there exist numbers $\mu > O$ and $\lambda > O$ such that: $\lambda\overline{x} \in T$ and $\mu\overline{y} \in P$. Without loss of generality,I can assume $\mu = \lambda = 1$. By definition of the relations $\geqq_s$, I can have $\overline{x}M_{\bar{\alpha}} \in K_P^*$ and $M_\alpha\overline{y}. \in K_T^*$. As K_P and K_T are the dual cones of K_P^* and K_T^* resp., I can get:

$\forall y \in K_P$ (and therefore, $\forall y \in P$), $\bar{x}M_{\bar{\alpha}}y \geqq O$, and

$\forall x \in K_T$ (and therefore, $\forall x \in T$), $xM_{\bar{\alpha}}\bar{y} \leqq O$

Therefore, $(\bar{x}, \bar{y})$ is a pair of minimax solutions to the game $(M_{\bar{\alpha}}, T, P)$, where $\bar{x}M_{\bar{\alpha}}\bar{y} = v(M_{\bar{\alpha}}, T, P) = O$. The fulfillment of (iii) need not be discussed. Let $\hat{x}$, $\hat{y}$, $\hat{\alpha}$) be an equilibrium solution, i. e., i) through (iii) of (DEFINITION 1') are fulfilled. Then clearly $\hat{x} \geqq_S o$, $\hat{y} \geqq_S o$ and $\hat{x}B\hat{y} > O$. By **XX** (resp. **XXI**) and due to $v(M_{\hat{\alpha}}, T. P) = O$, I can get $\hat{x}M_{\hat{\alpha}} \in K_P$(resp. $M_{\hat{\alpha}}\hat{y} \in -K_T^*$), i. e., $\hat{x}M_{\hat{\alpha}} \geqq_s O$ (resp. $M_{\hat{\alpha}}\hat{y} \leqq_S O$. }

(THEOREM 2)

[Let $\cdot M$ be a linear transformation of X into W, $M\cdot$ the dual transformation of Y into Z, $q \in W$. Exactly one of the systems of inequalities:

(i) $xM \geqq_S q$, $x \geqq_S o$:

(ii) $My \leqq_S O$, $qy > o$, $y \geqq_S o$;

has a solution——either (i) or (ii), but not (i) and (ii)].

Conclusions

* An Equilibrium Solution under Relaxed Assumptions:

In the following, (H 3) is replaced by

(H3) $x^+By^+ > O$.

Evidently, $x^+By^+ > O$ is the value of the game (B, T, P). Thus, (H3') corresponds to the assumption v (B) > O made by K. M. T. (i. e., v (B) denotes the value of the game with payoff matrix B, $S^m = \{x \in R_+^m \mid \Sigma_i x_i = 1\}$ and $S^n = \{y \in R_+^n \mid \Sigma_j y_j = 1\}$ as sets of strategies). By the same reasoning there exist α's fulfilling $\psi(\alpha) = o$. Moreover, there is a largest α for which $\psi(\alpha) = o$, called $\bar{\alpha}$, and one has $\bar{\alpha} > o$. So, in this case, for convenience, I can write

XXXV. $X^o := \{x \mid xM_{\bar{\alpha}} \geqq_S O, x \geqq_S o\}$

XXXVI. $Y^o := \{y \mid M_{\alpha}y \leqq_S o, y \geqq_S o\}$

Recall that

XXXVII. $\forall (x, y) \in X^o x Y^o$, $xM_{\bar{\alpha}}y = O$

This follows from $\psi(\bar{\alpha}) = o$. Now, I can assume that there exists no equilibrium solution at $\bar{\alpha}$, i. e.,

XXXVIII. $\forall (x, y) \in X^o x Y^o$, $xBy = O$ or, equivalently, $xAy = O$.

Then, due to **XXXVII**, **XXXVIII**, **XXIII** and **XXIV** and as $y^- \geqq y^+$) I can get

XXXIX. $\forall x \in X^o$, $xM_{\alpha}^j > O$ or $xM_{\alpha}^j < O \Longrightarrow y_j^+ = o$, $(\forall x \in X^o, xM_{\alpha}^j < O \Longrightarrow y_j^- = o)$.

Define

XL $J' = \{j \in J \mid \forall x \in X^o, xM_{\alpha}^j = O \text{ and } y_j^- > o\}$.

As $x^+By^+ > O$, I can get from **XXXIX**
XLI $\quad J' \neq \emptyset$.

For each $j \in J'$, define $q^j = (0 \ldots 1 \ldots 0)$ (n components with **1** in the jth position) and consider the following system:
XLII $\quad xM_{\bar{\alpha}} - q^j \geqq_S O \quad x \geqq_S o$.
By **XXXIX** and the definition of J', it is easy to find $\overline{y}^j \in K_P$, such that for each $j \in J'$, $x \in X^o \Longrightarrow (xM_{\bar{\alpha}} - q^j)\ \tilde{y}^j < o$:
hence,there exists no solution for **XLII**. Therefore,as a consequence of THEOREM 2, for each $j \in J'$ there exists a solution for the system:
XLIII $\quad M_{\bar{\alpha}}y \leqq_S O,\ q^j y > o,\ y \geqq_S o$.
In other word, for each $j \in J'$, there is a $\overline{y}^j \in Y^o$, $\overline{y}^j_j > o$, and therefore, using **XXXVII**, **XXXVIII**, and as $A \geqq 0$, I can get

XLIV. $\quad \forall j \in J',\ \forall x \in X^o,\ xA^j = O$.
I am now able to show that there exist $\overline{x} \in X^o$ and $\boldsymbol{\delta} > o$, such that

XLV. $\quad \overline{x}M_{\bar{\alpha}+\delta} \geqq_S O,\ \overline{x} \geqq_S O$.

which contradicts the definition of $\overline{\alpha}$ an therefore, proves the existence of an equilibrium solution.
To this end,define

XLVI $\quad J'' := \{j \in J \mid j \notin J',\ y_j^- > o\}$
$\qquad J''' := \{j \in J \mid y_j^- = o\}$

Obviously, $J' + J'' + J'''$.
If $J'' = \emptyset$, let $\overline{x}$ be any point in X^o and $\boldsymbol{\delta}$ be any positive number, then go to (＊).
If $J''' = \emptyset$, then by definition of J'' (and J') .
XLVII. $\quad \forall_j \in J'',\ \exists x^j \in X^o,\ (x^j M_{\bar{\alpha}})_j \neq O$.
By **XXXIX**, and since $y_j^- > o$, I must have $(xM_{\bar{\alpha}})_j \geqq O$ for each $x \in X^o$ and $j \in j''$; hence,
XLVIII. $\quad \forall j \in J'',\ (x^j M_{\bar{\alpha}})_j > O$.
Define $\overline{x}$ to be the centroid of $\{x^j \mid j \in J''\}$; then by **XLVIII** and by convexity of X^o, I can have
XLIX. $\quad \overline{x} \in X^o$ and $\forall j \in J'',\ (\overline{x}M_{\bar{\alpha}})_j > O$.
Let
L. $\quad \boldsymbol{\delta} := \mathrm{Min}\ \{(\bar{x}M_{\bar{\alpha}})_j / (\bar{x}A)_j \mid j \in J'',\ (\bar{x}A)_j > O\}$.
By the last observation, $\boldsymbol{\delta} > o$. And for each $j \in J$, I can have $\boldsymbol{\delta}(\overline{x}A)_j \leqq (\overline{x}M_{\bar{\alpha}})_j$ and,hence,

LI. $\forall j \in J''$, $(\bar{x}M_{\bar{\alpha}+\delta})_j = (\bar{x}M_{\alpha^-})_j - \boldsymbol{\delta}(\bar{x}A)_j \geqq O$.
($*$) For any $j \notin J''$, either $j \in J'''$, i. e., $y_j = o$, or $j \in J'$. In the latter case, by definition of J' and by **XLIV**, I can have $(\bar{x}M_{\bar{\alpha}})_j = O$ and $(\bar{x}A)_j$, and therefore,
LII $\forall j \in J'$, $(\bar{x}M_{\alpha+\delta})_j = O$.

Now, let y be any point in K_P; then

$$\bar{x}M_{\bar{\alpha}+\delta}\, y = \sum_{j\in j'''} (\bar{x}M_{\bar{\alpha}+\delta})\,\underbrace{{}_j y_j}_{=0} + \sum_{j\in j'} \underbrace{(\bar{x}M_{\alpha+\delta})_j}_{\substack{=0\\(\text{by}(52))}} y_j + \sum_{j\in J''} \underbrace{(\bar{x}M_{\bar{\alpha}+\delta})_j}_{\substack{=0\\(\text{if } J'' \neq \phi,\ \text{by } (50))}} y_j \geqq O.$$

With this, **XLV** is established, and the proof for the existence of an equilibrium solution is complete. A dual argument can be used for α, the minimal α giving $\psi(\alpha) = o$

I have shown how the theory of international economic trade under uncertainty can deal with rich structures of sectoral relationships across countries, without undermining the factor proportions basis for the world economy. It is now clear that, provided factor prices are equalized internationally, the factor proportions explanation of net economic trade flows is consistent with variable returns to scale, imperfect competition, and the presence of uncertainty. Every deviation from the traditional, deterministic, constant returns to scale paradigm, introduces new elements that affect gross trade flows, but there exists a wide variety of circumstances in which net trade flows preserve the basic features of the Heckscher-Ohlin insight. This insight has been shown to remain valid in the absence of factor price equalization in deterministic environments with constant returns to scale and perfect competition, and some form of imperfect competition with increasing returns to scale(see Helpman and Krugman1985)). Whether these results can be extended to stochastic environments remains an open issue.

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国際間経済取り引きに関する開放系均衡モデルの展開と吟味＝＝要約）

（*）：本稿での論証内容は次の通りである。
（1）生産性ギャップの確率タイプの変化の態様が一定のときの各国別の線型生産関係数の定義。
（2）消費と投資にかんするHomotheticな選好関数の定義。
（3）要素価格均等化を保証する一定の境界条件内での各要素量の各国別対比とそのチェック。（4）財貨と資産の自由な国際間取り引きの定式化。（5）確率タイプの変化を示さない生産量をクリアする産業部門の個数の算定問題の吟味。（6）確率タイプの生産性の変化が各国相互で完全な相関をとるとき、生産要素の個数は産業部門の個数に等しくなることにかんする論証。（7）本稿での論証作業のベースになっているのは、国際間の経済取引きの総量に関してBowen, LeameおよびSveikauskasの3人が共同開発した総量確定因子モデル設定のコンセプトである。

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